Progressive Services, Asymptotically Stagnant Services, and Manufacturing: Growth and Structural Change

by

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Abstract: Unlike almost all existing models of structural change, which treat services (*S*) as a single homogeneous or symmetric sector, our model sub-divides *S* into 'Progressive Services' (*PS*) and 'Asymptotically Stagnant Services' (*AS*), to better reflect the advent of the New Economy. A manufacturing (*M*) sector is also included, and we assume non-homotheticity of preferences between *S* and *M* consumption. An expanding-product-variety endogenous growth framework is adopted, and partially overlapping input sets across the three (sub-)sectors assumed. The interaction of these features results in a complicated but rich analysis, with different stages of growth generated endogenously: services which in due course become classified as Progressive first overtake *AS*, and then *M*, in innovation-driven productivity growth, consistent with post-World-War-II US experience. The *IT* revolution, and earlier phases of technological change, both influence and are influenced by sub-sectoral evolution, such innovation and structural change thus forming integral features of an endogenous, dynamically evolving growth pattern. The socially optimal growth pattern differs qualitatively from the private, and exhibits a novel, endogenous-growth-related feature, 'Initial Condition Dependence'. Differentiated, time-varying *R&D* subsidies to the different input sets are optimally called for.

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1. INTRODUCTION

Economic growth has been accompanied by major structural changes in advanced economies. In the US, for example, over the 1948–2000 period the share of the private Service Sector in current-value GNP has risen steadily from 50.1% to 71.1%, while that of private Manufacturing has fallen from 30.4% to 14.9% (US Bureau of Economic Analysis (2011)). Similarly, over the same period the employment share of Manufacturing in the US fell from 26.5% to 13.9%, and that of Services increased from 48.8% to 69.3% (*ibid.*).[1] Data for other OECD countries paint a similar picture (Aarnio (1999), Tables I and II).

Behind the scenes of these developments are other, less readily discernible but equally significant, changes. With the advent of the so-called 'New Economy' and the *ICT* (information and communications technology) revolution, the traditional view of Services as a laggard in productivity growth, epitomized in 'Baumol's Disease' (Baumol (1967)), has given way to a more differentiated view. As discussed further in Section 2, productivity growth in a number of service industries that are heavy *ICT* users has *exceeded* average productivity growth in Manufacturing in recent years, and indeed Edward Wolff has in 2002 proposed a division of Services into 'progressive' and 'stagnant', or 'asymptotically stagnant' (Baumol *et. al.*, 1985, 1989; see also Baumol, 2002), sub-sectors.

With *ICT* being a key part of the story, the focus naturally shifts next to *R&D* and technological advances, in both *ICT*- producing and using industries. There are notable cross-sector linkages – a heavy *ICT*-using service industry, for example, such as banking and finance, will avail of advances in *both* software and hardware. Moreover, a more satisfying treatment would view spending on *R&D*, at sectoral as well as aggregate levels, as endogenous too, being both driven by and contributing to the structural changes just described.

[1] Kozicki (1997, *p*. 37) suggests that one reason for the rising shares of services in employment and GNP could be increased outsourcing of service functions by manufacturing firms. On the other hand, 'many service activities (such as finance and leasing), and..their supporting *R&D*, have migrated from the service sector to the manufacturing sector' (Gallaher *et. al.* (2005), *p*. 2-9). Empirical findings are discussed further in Sections 2 and 3.2.2 (c) below.

Our purpose in this article is, accordingly, to study the foregoing changes in sectoral output, employment, productivity growth and *R&D* in a unified framework in which all of these are endogenous. In line with Wolff, we divide Services into two sub-sectors, 'progressive' ('New Economy') and 'asymptotically stagnant' – a distinction that formal models of structural change have hitherto not made.[2] Notwithstanding the emphasis in the above on supply-side developments, we also show that the demand side – in particular, the introduction of non-homotheticity of preferences – is a necessary part of the story. Our analysis produces a *stages-of-growth* outcome in which, under reasonable parameter configurations, productivity growth in Progressive Services *PS* (strictly, services which in due course become classified as progressive) first endogenously overtakes that in Asymptotically Stagnant Services *AS*, and subsequently overtakes that in Manufacturing – consistent with the historical experience.

For tractability, we work with the expanding-product-variety (*EPV*) framework of Romer (1990) and Grossman and Helpman (1991; *RGH* henceforth), extended to a multisectoral setting. A fundamental difference between our model and previous multisectoral ones, however, is that we allow for *partial* overlap of input variety sets across the three (sub-)sectors of the model, whereas earlier models, as discussed further below, assume either complete overlap or completely disjoint input variety sets. A major consequence of our treatment is that corner solutions, in which *R&D* in one or more input variety sets completely ceases, figure prominently in our analysis: these complicate the analysis considerably, and at the same time the endogenously evolving pattern of corner solutions plays a key role in generating our distinctive stages-of-growth outcome.

In Section 2, we review changing sectoral productivity patterns and their link in recent years to the *ICT* revolution, and also discuss prior studies of structural change. Section 3 sets out and solves the model, showing that its dynamic adjustment path does indeed resolve into three growth stages. Section 4 shows that there are

[2] Buera and Kaboski (2006) distinguish sectors by skilled-labour-intensity, with less skill-intensive services progressively transiting from market to home production as incomes and economy-wide skill levels rise. Their skill-intensive sectors straddle some progressive services in our classification, such as banking, and some asymptotically stagnant ones, such as health and education. They also assume constant**,** exogenous Total Factor Productivity Growth, whereas endogenizing this, at sub-sectoral levels, is one of our major concerns. See also Buera and Kaboski (2007).

interesting and significant qualitative differences between the socially optimal growth path and the private one, and also examines the policies required to achieve the former. Finally, Section 5 concludes.

2. RECENT DEVELOPMENTS

Prior to discussing the empirical evidence, it is useful to clarify some conceptual and interpretational issues. First, as pointed out by Stiroh (2002, *p*. 1562) and others, in the New Economy era labour productivity growth (*LPG*) is a more revealing measure at the industrial or sectoral levels than *MFPG* (multifactor productivity growth), since the former includes both the latter and the benefit from *ICT*-capital-deepening in *ICT*-using industries or sectors induced by rapid declines in recent years in the prices of *ICT* capital items due to technological advances. As Stiroh puts it, 'the primary effect of *IT use* is likely through traditional capital deepening channels' (emphasis added) – which the labour productivity measure captures.

The foregoing suggests that within Services it would be useful to distinguish between *ICT*-Producing, *ICT*-Using, and Non-*ICT* Services (*ICTPS*, *ICTUS*, *NICTS* henceforth), and similarly for Manufacturing. To the author's knowledge, the only study which does this is van Ark, Inklaar, and McGuckin (2003, *AIM* henceforth). Table 1 below, extracted from Table 3.1 of *AIM*, provides their estimates of growth of US labour productivity (defined as value-added per employed person) for these sub-sectors for 1990-1995 and 1995-2000 (post-2000 developments are discussed below). Our focus will be on the *ICTUS* and the *NICTS* industries: as earlier indicated, there are notable cross-sector linkages, with *ICT*-Producing Manufacturing (*ICTPM*), for example, contributing significantly to *ICTUS* over time. The remarkable productivity growth rates in *ICTPM* recorded in Table 1 would certainly have contributed considerably to productivity growth elsewhere in the economy.

Thirdly, productivity growth measurements over time are fraught with pitfalls, which may help to explain the two negative numbers in Table 1 in the earlier period. It has often been argued that there are significant adjustment costs in *ICT*-adoption, which, if the underlying momentum of productivity growth in *ICT*-Using Manufacturing (*ICTUM*) during that period was low, could have sufficed to push the actual productivity growth into negative territory. As for Services, there has been a vast, and continuing, differentiation of these, which hedonic measures, that are still being developed and implemented, do not fully capture, particularly for earlier periods.[3] Much of this differentiation – such as the growth of Internet banking, and the expansion of cable TV services, both through increases in the number of networks and in programming expenditures by existing networks (Wildman, 2003) – has been facilitated by advances in *ICT* (see also McGuckin and Stiroh (2001)), while other forms of differentiation are less 'high-tech' but economically significant nonetheless, as Gadrey (2002) insightfully argues.[4] These difficulties do not totally invalidate the numerical calculations, but point to the need for a more discriminating approach to interpreting them, looking at (sub-)sectoral aggregates as well as industry detail as necessary.

We turn next to the empirical findings. Table 1 shows that there has been a remarkable acceleration in *LPG* in *ICTUS*, from 1.9% *p.a.* in 1990-1995 to 5.4% *p.a.* in 1995-2000. It has overtaken *LPG* in Non-ICT Manufacturing (*NICTM*) over this decade, and also exceeds *LPG* in *ICTUM*. (Estimated *LPG* in *ICTUM* in the earlier period was slightly negative, probably reflecting the influence of the adjustment costs mentioned earlier, and in the latter period was close to that of *NICTM*.) On the other hand, no such overtaking has been accomplished by *NICTS*, whose *LPG* was slightly negative in the earlier period, likely reflecting the measurement difficulties alluded to earlier, and low in the latter period. The most remarkable *LPG* has occurred in *ICTPM*, mainly computers, semiconductors, communications equipment, and fibre

[3] Bosworth and Triplett (2003 (*BT* henceforth), discussed further below) make use of the improved Producer Price Index series compiled by the US Bureau of Labour Statistics (BLS), but the coverage of service industries is not complete (BLS, 2007), and Lum, Moyer, and Yuskavage (2000) point out that 'problems persist in defining the gross output of many services industries', an assessment that *BT* share (*pp.* 35 *ff.*).

[4] Taking supermarkets as an example, Gadrey asks (*p.* 40), 'do we get the same output when, for the same basket of goods bought, we get our purchases bagged, ..carried to our car on request, when we have 50 percent more varieties of goods..to choose from, when opening hours per week are twice as long, ..when more scanning systems save our time at the checkout counter (*etc.*)'? He adds that, in view of the resulting increased labour requirements, 'if this more complex, service-based approach is adopted, it can be shown that, during the 1980s, the average US supermarket clearly increased its performance in terms of quality and quantity of services delivered, *leading to a decrease in its productivity as measured in the traditional way* (which is simply based on 'sales at constant prices')' (emphasis added). Gadrey focuses on supermarkets, while Varian (2004) has additionally identified 'hotels, health, education and entertainment' as 'all examples where customers tend to perceive that more labour is associated with higher quality'.

optics, namely 15.1% in 1990-1995 and 23.7% in 1995-2000, while that in *ICTPS* has been positive but much slower.

Clearly, then, a distinction needs to be drawn between *ICTUS* – which can be viewed as corresponding to our *PS* – and *NICTS*, our *AS*.[5] On the other hand, for analytical, model-building purposes, the empirical findings suggest that one may conveniently combine *ICTUM* and *NICTM* into a single *M* variable without affecting the results qualitatively.[6] Lastly, we shall treat *PS*, *AS*, and *M* as drawing for production purposes upon, *inter alia*, sets of produced intermediate inputs, without seeking to distinguish whether the latter are produced within their own sub-sectors or sectors, or outside, and for notational convenience the aggregate of *PS* and *AS* is denoted by *S*.

The foregoing discussion also indicates that *R&D* spending at the sectoral or sub-sectoral levels is not necessarily a good indicator of productivity trends at those levels. Such spending may be fairly closely correlated with *MPFG*, but not with *LPG*. Consistent with the observation by McGuckin and Stiroh (*fn.* 6), *BT* find that '(m)ost of the effect of *IT* capital deepening in the U.S. economy in recent years shows up in its contribution to labor productivity growth in the services industries' (*p.*

[5] *AS* are an extension of Baumol's (1967) original notion of stagnant sectors, which formed the basis of Baumol's disease. As concisely described by Baumol (2002), stagnant sectors are assumed to employ only labour (assumed to exhibit zero productivity growth), whereas *AS* employ labour, or the product of the stagnant sector, as well as an input from a dynamic sector in which productivity grows exponentially. Under mild restrictions, it is established that '(t)he behaviour of the average cost of an asymptotically stagnant sector will approach, asymptotically, that of the stagnant sector from which the former obtains some of its inputs' (*ibid.*, *p.* 155). It turns out, *inter alia*, that Baumol's proposition regarding the behaviour of average costs does not hold in our *EPV* framework, but a somewhat similar result does (*fn.* 24 below). Wolff's (*op. cit.*, *p.* 5) division of service industries into *PS* and *AS* is motivated by his finding that 'there are extreme differences in amenability to productivity growth' across service industries.

[6] LPG for *ICTUM* was in fact slightly below that for *NICTM* during 1995-2000, which may reflect the lingering effects of adjustment costs, as well as (*AIM*) reallocations of labour across industries within each sub-group (this may of course have been an influence for other sub-groups and periods as well). *AIM* use a median cut-off to distinguish *ICT*-using and non-*ICT* (strictly, less *ICT*-using) industries, but do not provide a ranking of industries in the *ICT*-using category. McGuckin and Stiroh (p. 295) allude, however, to 'the well-documented fact that high-tech investment is heavily concentrated in non-manufacturing industries like banking and business services, e.g., Stiroh (1998) and Triplett (1999)'.

22), highlighting again the significance of such capital-deepening as a distinct contributor to *LPG*, separate from, and in addition to, sectoral *R&D* spending.

Taking a longer-term perspective yields another interesting result. On the basis of 1947-73 data, Wolff (2002) has classified Wholesale and Retail Trade and FIRE (Finance, Insurance, and Real Estate) as stagnant services, and Transportation and Electric, Gas and Sanitary Services as progressive services[7] – the precise opposite of the classifications by *AIM* based on later data! This earlier period is prior to the intensive *ICT*-capital-deepening era, for which *MFPG* is the more appropriate measure, and Wolff's estimates, in conjunction with 1947 value-added weights from the Bureau of Economic Analysis' 2007 National Accounts database indicate that *MFPG* in the former two of these industries during that period was 1.21% *p.a.*, and for the latter two was 1.90% *p.a.*[8] Over time, thus, Wholesale and Retail Trade and FIRE have first overtaken other service industries in productivity growth, and, in recent years, overtaken *M* as well – and our model will have to account for both sets of phenomena.[9]

What about the situation after 2000?[10] Jorgensen *et. al.* (2007), in studying the 2000-2005 period, note that after the dot-com crash of 2000 the rate of decline of *IT* prices slowed, relative to the 1995-2000 period. 'Investment in *IT* equipment and software slowed, but remained strong relative to the pre-1995 period due to the low prices already in place' (*p*. 2). Interestingly, the authors find that '(r)educed *TFP* growth in *IT* production was more than offset by a sharp rise in *TFP* growth in the *IT*-using industries, principally in services'. This helped significantly in sustaining

[7] Wolff suggests (*p*. 21) that productivity comparisons in the 1970's are affected by the 1973 oil price shock, and in the 1980's by the fact that services output 'is becoming increasingly more difficult to measure'.

[8] Wolff lumps all other services into a 'General Services' category, some of which are later classified by *AIM* as *ICT*-using, and some as non-*ICT*-using: we ignore these, as *MFPG* data on them separately are not provided by Wolff.

[9] Wolff provides estimates of *MFPG* in the earlier period for durable and nondurable manufacturing, but his figure of 0.4 for the latter appears to be a significant underestimate (compare, for example, BLS figures reported in Bagnoli (1997, Table IX), although for somewhat differing time intervals). The estimate for the most closely-matched time-interval to Wolff is by Lysko (1995, Table 1), who estimates US manufacturing *MFPG* over 1956-73 to be 2.8% *p.a.* – above the service sub-sector estimates reported above.

[10] I am deeply indebted to an anonymous referee for suggesting that I consider the issues discussed in this and the next paragraph, and in *fn*. 26 below.

overall US *LPG* during 2000-2005, which actually accelerated beyond its level during 1995-2000 (although other factors, such as accelerated capital-deepening, also played a role in this (*ibid*.)). In a separate study Sharpe (2004) insightfully observes (*pp*. 21-22) that 'the full productivity-enhancing effect of *ICT*s may not have been fully realized (during 1995-2000) because of the organizational changes needed to effectuate these gains. It is thus possible that because of the lags required for the effective use of *ICT*s, only since 2000 has the full impact of *ICT*s on productivity been realized', and the preliminary data he studied indicated that – consistent with Jorgensen *et. al.*'s findings – '(t)his may be particularly true in service industries'. He also observes that technological, including organizational, improvements frequently entail ongoing, as well as lagged, capital investments (*pp*. 22-23), and one may add that such improvements could well be a continuing process, as better versions and new applications of *IT* equipment and software continue to be developed.[11]

The financial crisis that commenced in mid-2007 has raised the issue of how accurate banking-sector output, and hence productivity, measurements really are. In an excellent study, Basu, Inklaar, and Wang (2011) formally apply 'the idea from financial intermediation theory that the main service provided by banks in making loans is reducing asymmetric information between borrowers and lenders through screening and monitoring. Instead of receiving an upfront fee for these services, an optimizing bank can charge a higher interest rate than the rate available on a market security with otherwise the same risk attributes' (*p*. 227). The correct measure of bank loan service output should thus, they argue, be based on the difference between

[11] Another fairly contemporary issue is globalization, and whether the offshoring of manufacturing production to low-wage countries such as China is a major contributor to structural changes in the US. As indicated in our Introduction, however, the structural changes we focus on are long-term in nature, and have been in progress since 1948, well before the current era. Moreover, even in recent years, there are sharp divergences between manufacturing output and employment trends. For example, between 1990 and 2000 manufacturing employment in the US contracted by about 3% in absolute terms, while real goods output (overwhelmingly manufacturing) *increased* by 59.1% (US Bureau of Economic Analysis (2011)), quite contrary to what one would expect if offshoring were a major economic force. Productivity growth and a shift in the US industrial structure towards increasingly more technology-intensive and high-value-added products (many of which are exported), account for these divergent trends, and even currently 'the fact is that the US remains the largest manufacturing economy in the world by a healthy margin' (Huether (2010)). There is a continuing controversy as to the effect of offshoring on skilled-unskilled wage differentials, which is concisely reviewed in Hillebrand *et. al.* (2010).

the actual bank lending rate and the rate on the corresponding (risky) market security, instead of the difference between the lending rate and a risk-free rate, as is the current practice.[12] Their Figure 4 shows indeed that market risk-adjusted interest rates rose sharply in the last two quarters of 2007, the last period of their sample, (further) reducing their measure of banking sector output. One of the study's co-authors, Robert Inklaar, has, however, indicated (in a kind personal communication) that, while their adjustment for risk affects measurement of the *level* of bank output each year, 'in earlier exercises this did not have a major effect on overall bank output *growth*' (emphasis added), so that, at least prior to 2007, one may indeed include banking in the *PS* set.[13]

A number of studies of structural change have appeared in recent years, but omit one or more of the features discussed earlier. Except for Buera and Kaboski (*fn.* 2 above) all treat *S* as a single, homogeneous sector (or as a sector all of whose constituent industries have symmetric production structures), and do not disaggregate it into *PS* and *AS*. Xu (1993), Echevarria (1997), Kongsamut *et. al.* (2001), and Ngai and Pissarides (2007) assume exogenous technical progress, at the same or differing rates across sectors, while Stokey (1988) allows for learning-by-doing in one sector (manufacturing) but not in another (agriculture).

Meckl (2002), Klenow (1996), and Acemoglu and Guerrieri (2006) adopt the *EPV* framework of *RGH*, as we do: *RGH* assume a single final-goods sector, whereas these models are multisectoral. (Acemoglu-Guerrieri and Ngai-Pissarides also allow for physical capital accumulation, which we abstract from.) Meckl assumes identical intermediate-input sets across all sectors, helping to ensure that relative final output prices are constant over time, while Klenow and Acemoglu-Guerrieri assume completely disjoint sectoral input sets, as well as homothetic tastes (they also impose further specifications that rule out corner solutions, notwithstanding the disjointness).

[12] The risk premium of the market security's interest rate over the risk-free rate should be treated as 'part of the borrowing firms' cost of capital and hence as part of households' capital income, their compensation for bearing risk' (*ibid*.).

[13] In addition, with reference to more novel bank activities, such as securitization and the underwriting of derivatives, Prof Inklaar has informed me that available statistics do not permit a rigorous treatment of these, and also that the information available suggests that, at least till recently, they generate 'a fairly small share of bank revenue,…so any impact on overall bank output would also be small. That doesn't mean derivatives are unimportant, just not a major factor in bank output trends'.

Either treatment of intermediate-input sets simplifies the analysis considerably. However, our earlier discussion clearly indicates that *ICTUS* draws more heavily on *ICT* inputs than *NICTS* or *M*, so that the commonality of inputs would tend to be greater (although still not complete) between the latter two than with the first-mentioned sub-sector.[14] Thus, the assumption of *partial* overlap of input sets, as detailed more precisely below, appears most appropriate. As will be seen, it is the combination of this and the assumption of taste non-homotheticity that yields our distinctive stages-of-growth outcome.

3 THE MODEL AND ITS SOLUTION

3.1 *Model specification*

3.1.1 Production. We adopt a three-(sub-)sector *EPV* framework, and begin with the *M* sector:

(1) $Y_M \;=\; A_M L_M^{1-\beta_1-\beta_2} D_{M1}^{\beta_1} D_{M2}^{\beta_2}, \quad \beta_1, \beta_2 > 0,\; \beta_1 + \beta_2 < 1,$

(2) $D_{M1} \;=\; [\int_0^{n_{M1}} x_{M1j}^{\alpha_{M1}} dj]^{1/\alpha_{M1}}, \quad 0 < \alpha_{M1} < 1,$

(3) $D_{M2} \;=\; [\int_0^{n_2} x_{M2j}^{\alpha_2} dj]^{1/\alpha_2}, \quad 0 < \alpha_2 < 1,$

where Y_M denotes output of final manufactured goods, L_M labour input in *M* (capital is abstracted from in the model), and D_{M1} and D_{M2} are two composite sets of produced intermediate inputs with x_{Mij} denoting the amount of input *j* employed in set *i* (*i* = 1, 2). D_{M1} comprises inputs such as automobile transmission systems that are specific to *M*, while D_{M2} comprises inputs such as word processing programs and office furniture which are employed in both *M* and *S*: as will be seen this distinction, apart from being descriptively appropriate, has significant analytical implications. The measures of the varieties of intermediate inputs available in each set, n_{M1} and n_2, grow over time as a

[14] Guo and Planting's (2000) input-output (*I-O*) analysis for the US shows that 'there is substantial variation in linkages between industries' (p. 12) in each *I-O* table estimated at five-year intervals in the 1972-96 period (the pattern of linkages also changes gradually). There is not an exact concordance between Guo and Planting's consolidated 16-industry classification and *AIM*'s, but Table 3a in the former, on 'total output multipliers' (a measure of backward interindustrial linkages) indicates that these linkages are almost without exception higher for industries we classify as belonging to *M* and *AS*, than those belonging to *PS*: moreover, the more disaggregated, two- and three- digit industries that *PS* industries more heavily draw from are as indicated above likely to be quite different from those that industries in the other groups heavily draw from.

result of technological progress (modeled below), as does a corresponding measure in *S*.

Specification of the *S* sector is more involved, and we begin with

$$(4) \quad Y_S \;=\; A_S S_1^{\theta} S_2^{1-\theta}, \quad 0 < \theta < 1,$$

where S_1 and S_2 are indices of output of *PS* and *AS* respectively: below, we adopt a utility function for the representative agent that permits these outputs to be consistently aggregated into the foregoing overall index of service sector output (and consumption), Y_S. It is convenient to discuss the second sub-sector first:

$$(5) \quad S_2 \;=\; A_{S2} L_S^{1-\omega} D_{S2}^{\omega}, \quad 0 < \omega < 1, \quad \text{where}$$

$$(6) \quad D_{S2} \;=\; [\textstyle\int_0^{n_2} x_{S2j}^{\alpha_2} dj]^{1/\alpha_2} .$$

AS thus requires inputs of labour and the same measure n_2 of 'common' inputs as set D_{M2}: for simplicity, we assume the same productive parameter α_2 in both, although the quantities of individual inputs employed can of course differ between D_{M2} and D_{S2}. The set of inputs forming D_{M2}, D_{S2} is henceforth denoted as set 2.

For *PS* we have

$$(7) \quad S_1 \;=\; [\textstyle\int_0^{n_{S1}} x_{S1j}^{\alpha_{S1}} dj]^{1/\alpha_{S1}} ,$$

indicating that this sub-sector utilizes specialized inputs x_{S1j}. Since S_1 is, following our earlier discussion, an *ICT*-intensive sector the x_{S1j}'s would largely be drawn from analogues in our model of *ICTPM* and *ICTPS*[15] (note that we are not restricting the produced inputs into any sub-sector to come only from intermediate-input-producing manufacturing – different from *M*, which refers to final manufacturing output – or only from intermediate-input-producing services (different from *S*)). Also, following from our earlier discussion, apart from the literal input interpretation employed here, increases in n_{S1} over time could equally well be viewed as reflecting the vast differentiation of final services offered to consumers, such as the expansion of services offered by Internet banking, supermarkets (part of *PS*, using *AIM*'s classification), and many other service providers.

[15] *ICTPM* and *ICTPS* are final outputs, presumably because they significantly comprise output of long-lived capital products, whereas our model abstracts from capital goods. Table 11 of Yuskavage (1996) indicates that the pronounced long-run declining share of manufacturing and rising share of services in US GDP hold even if *ICTPM* and *ICTPS* are excluded, and for tractability we assume that the *ICT* items used in S_1 comprise short-lived intermediate inputs, both manufactures and services.

Algebraic convenience motivates some asymmetries in the foregoing formulation. In principle, *PS* could also employ labour, as well as some common inputs, and *AS* could also employ some specialized inputs. These would not change our qualitative results, and for algebraic ease are abstracted from. What *is* important, obviously, is that the input-intensities in the three (sub-)sectors and the α_k (k = $M1$, 2, $S1$) parameters not all be identical, and our previous discussion, including Baumol's observation that *AS* employs relatively more direct labour than *PS*, clearly points to this. As will be seen below, differing configurations of input-intensities and parameter values are a major source of endogenous, differential (sub-)sectoral rates of productivity growth in the model.[16]

Producers of M, S_1 and S_2 are assumed to be price-takers in output and input markets, and likewise for consumers (in all output markets). By suitable choice of A_M and A_{S2} we may then obtain

(8) $p_M = w^{1-\beta_1-\beta_2} p_{M1}^{\beta_1} p_2^{\beta_2}$,

(9) $p_{M1} = [\int_0^{n_{M1}} p_{M1j}^{-\alpha_{M1}/(1-\alpha_{M1})} dj]^{-(1-\alpha_{M1})/\alpha_{M1}}$,

(10) $p_2 = [\int_0^{n_2} p_{2j}^{-\alpha_2/(1-\alpha_2)} dj]^{-(1-\alpha_2)/\alpha_2}$,

(11) $p_{S1} = [\int_0^{n_{S1}} p_{S1j}^{-\alpha_{S1}/(1-\alpha_{S1})} dj]^{-(1-\alpha_{S1})/\alpha_{S1}}$,

(12) $p_{S2} = w^{1-\omega} p_2^{\omega}$,

where p_M, p_{S1}, and p_{S2} are the output prices of M, S_1, and S_2, w is the wage rate, p_{M1} and p_2 are the imputed prices of composite inputs $M1$ and 2, and the 'j' subscript in equations (9)-(11) denotes the price of the individual item in the corresponding input index. One may also easily obtain

(13) $L_M = (1-\beta_1-\beta_2) p_M Y_M / w$,

(14) $D_{M1} = \beta_1 p_M Y_M / p_{M1}$

(15) $D_{M2} = \beta_2 p_M Y_M / p_2$

(16) $L_S = (1-\omega) p_{S2} S_2 / w$,

[16] Echevarria and Kongsamut *et. al.* (1997) adduce data indicating that labour's income share in S is lower than in M: their findings are not directly applicable here, since they examine labour's share in value-added, which is trivially unity in our model. Labour's share in total output is clearly lower for *PS* than M in our model, and, depending on relative output weights and parameter values, the same may or may not be true for S as a whole, relative to M.

(17) $D_{S2} \;=\; \omega p_{S2} S_2 / p_2$.

Cost-minimization by final goods producers yields the following derived demand functions for individual input items in the three sub-sectors:

(18) $x_{S1j} \;=\; S_1 p_{S1j}^{-\varepsilon_{S1}} p_{S1}^{\varepsilon_{S1}}$,

(19) $x_{M1j} \;=\; D_{M1}\, p_{M1j}^{-\varepsilon_{M1}} p_{M1}^{\varepsilon_{M1}}$,

(20) $x_{2j} \;=\; x_{M2j} + x_{S2j} \;=\; (D_{M2} + D_{S2})\, p_{2j}^{-\varepsilon_2} p_2^{\varepsilon_2}$,

with $\varepsilon_k = 1/(1-\alpha_k)$, $k = S1, M1, 2$, thus being the corresponding price-elasticities of demand.

Producers of individual intermediate inputs are assumed to be monopolistically competitive, and production of each x_{kj} requires only labour:

(21) $x_{kj} \;=\; L_{kj}$, $\quad k = S1, M1, 2$.

Thus, the profit-maximizing price charged for each x_{kj} is

(22) $p_{kj} \;=\; w/\alpha_k$, $\quad k = S1, M1, 2$.

(The *numeraire* of the model is introduced below.)

3.1.2 *R&D*. Letting L_{rk} denote labour devoted to *R&D* in group *k*, and dots denote time (*t*) derivatives, we assume, analogously to Romer,

(23) $\dot{n}_k / n_k \;=\; L_{rk} / a$, $\quad k = S1, M1, 2$

i.e. '*a/n* (a/n_k in our case) units of labour are needed to develop a new variety' (Grossman and Helpman, *p*. 118): endogenous growth therefore becomes possible. We thus distinguish *R&D* by the input group in which it occurs, although for convenience the research productivity parameter *a* is assumed to be the same across groups. Total labour employed in *R&D*, L_r, is given by

(24) $L_r \;=\; L_{rS1} + L_{rM1} + L_{r2}$.

It is assumed that the inventor of each new intermediate input enjoys a perpetual monopoly right over its production and sale – he may either, as assumed here, also be its producer, or equivalently charge a royalty for the use of his invention to competitive producers of the input (Barro and Sala-i-Martin (2004, *p*. 216)). Since firms face symmetric demand and production conditions within each group *k*, we may set $x_{kj} = x_k$ for all *j*, and likewise for all other firm variables. *R&D* costs are financed by issuing equity, the value of which for each firm in group k is denoted by v_k (equal to the discounted value of the firm's future profits). The no-arbitrage condition holds:

$$(25) \quad \dot{v}_k \;=\; rv_k - \pi_k, \quad k = S1, M1, 2$$

where r is the interest rate (to be determined, bonds and equities being perfect substitutes under certainty), and profits π_k for any firm in group k are

$$(26) \quad \pi_k \;=\; (p_k - w)x_k, \qquad k = S1, M1, 2.$$

Free entry is also assumed, so that if $\dot{n}_k$ is positive,

$$(27) \quad v_k \dot{n}_k \;=\; wL_{rk}, \qquad k = S1, M1, 2$$

which implies, from (23)

$$(28) \quad v_k \;=\; wa/n_k.$$

If the left side of (28) is less than the right, it is easily shown that $\dot{n}_k$ must equal 0. Whether $\dot{n}_k$ is positive or 0, it will be seen below that n_k and v_k can evolve quite differently over time for the different k's, which is a major complicating feature – and a major focus – of the analysis.

3.1.3 Consumers. Equities are the only asset in positive net supply, and their total value V is given by:

$$(29) \quad V \;=\; n_{S1}v_{S1} \;+\; n_{M1}v_{M1} \;+\; n_2 v_2.$$

Capital being abstracted from, consumption of each final product equals its output:

$$(30) \quad C_M \;=\; Y_M,$$

$$(31) \quad C_i \;=\; S_i \quad (i = 1, 2), \text{ and hence,}$$

$$(32) \quad C_S \;=\; Y_S$$

(assuming a Cobb-Douglas utility sub-aggregate for S). By suitable choice of A_S we then obtain the price index p_S of the overall services bundle C_S:

$$(33) \quad p_S \;=\; p_{S1}^{\theta} p_{S2}^{1-\theta}.$$

We assume a constant population of unit mass, and the representative agent, with a unit labour endowment, seeks to maximize

$$(34) \quad U \;=\; \int_0^\infty e^{-\rho t}\left[\frac{C_{Mt}^{1-\sigma}-1}{1-\sigma} + \tau \ln C_{St}\right]dt, \quad \sigma > 1,$$

subject to

$$(35) \quad \dot{V}_t \;=\; r_t V_t \;+\; w_t \;-\; (p_{Mt}C_{Mt} + p_{St}C_{St}),$$

where ρ is the rate of time preference. The instantaneous utility function in (34) is Houthakker's (1960) direct addilog function (specialized to the logarithmic form for the sub-utility function for C_{St}), which Ogaki (1992) has successfully implemented in studying non-homothetic preferences, this being another key issue we wish to focus on: it also has the merit that each sub-utility function is of the constant intertemporal elasticity of substitution form, which helps to ensure that the economy converges to a path on which sectoral and sub-sectoral growth rates are constant, although not necessarily equal.[17]

3.1.4 Market-Clearing Conditions. In addition to (30)-(32) at each t we have

(36) $$L_{Mt} + L_{St} + \int_0^{n_{S1}} L_{S1jt}dj + \int_0^{n_{M1}} L_{M1jt}dj + \int_0^{n_2} L_{2jt}dj + L_{rt} = 1,$$

and the market-clearing condition for each x_{kj} (equations (18)-(21)). The model is now complete, and by Walras' Law one equation is not independent: one can for example derive (35) from the other equations of the model.

3.2 *Solving the model*

3.2.1 Consumer Optimization and Instantaneous Equilibrium. The Hamiltonian of the consumer's optimization problem is

(37) $$H_t = \frac{C_{Mt}^{1-\sigma} - 1}{1-\sigma} + \tau \ln C_{St} + \lambda_t [r_t V_t + w_t - (p_{Mt}C_{Mt} + p_{St}C_{St})],$$

λ_t being the co-state variable. Necessary conditions for optimality are

(38) $$\frac{\partial H_t}{\partial C_{Mt}} = C_{Mt}^{-\sigma} - \lambda_t p_{Mt} = 0,$$

(39) $$\frac{\partial H_t}{\partial C_{St}} = \tau C_{St}^{-1} - \lambda_t p_{St} = 0,$$

(40) $$\dot{\lambda}_t = (\rho - r_t)\lambda_t,$$

[17] The expression for the expenditure elasticity of the first good (M) given in Ogaki reduces in our specification to $[\varsigma_{1t} + \sigma(1-\varsigma_{1t})]^{-1}$, where ς_{1t} is the share of $p_{Mt}C_{Mt}$ in total consumption ($p_{Mt}C_{Mt}$ + $p_{St}C_{St}$), while that for S is σ times the elasticity for M. With $\sigma > 1$, the elasticity for M is less than, and that for S greater than, unity. Empirical evidence in support of a higher elasticity for S than M – although in the context of cross-sectional analyses based on Deaton and Muellbauer's Almost Ideal Demand System – is provided in various studies of the DEMPATEM project (see, for example, Schmitt (2004), Blow (2004), and Gardes and Starzec (2004) for the US, UK, and France respectively). Ogaki arrived at the same finding, for selected S and M items. We discuss alternative utility-function specifications in Section 3.2.2(c) below.

and we also have the transversality condition

(41) $\quad Lim_{t\to\infty} e^{-\rho t}\lambda_t V_t \quad = \quad 0.$

Our characterization of C_{St}, the services aggregate, also implies

(42) $\quad p_{S1t}S_{1t} \quad = \quad \theta p_{St}C_{St}$, and

(43) $\quad p_{S2t}S_{2t} \quad = \quad (1-\theta)\,p_{St}C_{St}$.

It is easily verified that the Mangasarian sufficiency conditions for a maximum (Chiang (1992)) are satisfied, and for the time being we assume the existence of a growth path satisfying the necessary and sufficient conditions: conditions for existence are presented below. From (39), nominal spending on S, E_{St}, is

(39′) $\quad E_{St} \quad = \quad p_{St}C_{St} \quad = \quad \tau / \lambda_t$

We adopt the following normalization (*numeraire*):

(44) $\quad E_{St} \;\equiv\; 1 \quad$ for all t.

(Grossman and Helpman normalized total consumer spending to unity in their one-final-good model.) From (39′) this implies

(45) $\quad \lambda_t \quad \equiv \quad \tau$, all t,

which from (40) yields

(46) $\quad r_t \quad \equiv \quad \rho$, all t.

We then also have,

(47) $\quad C_{Mt} \quad = \quad \tau^{-1/\sigma} p_{Mt}^{-1/\sigma} \quad = \quad Y_{Mt}$

(48) $\quad C_{St} \quad = \quad p_{St}^{-1} \quad = \quad Y_{St}$.

We may also substitute the solutions for the various p_{kj} from (22) into (9)-(11) and we then obtain, using earlier equations as well:

(49) $\quad p_{M1t} \quad = \quad w_t \alpha_{M1}^{-1} n_{M1}^{-(1-\alpha_{M1})/\alpha_{M1}}$

(50) $\quad p_{2t} \quad = \quad w_t \alpha_2^{-1} n_2^{-(1-\alpha_2)/\alpha_2}$

(51) $\quad p_{Mt} \quad = \quad w_t \alpha_{M1}^{-\beta_1} n_{M1}^{-\beta_1(1-\alpha_{M1})/\alpha_{M1}} \alpha_2^{-\beta_2} n_2^{-\beta_2(1-\alpha_2)/\alpha_2}$

(52) $\quad Y_{Mt} \quad = \quad \tau^{-1/\sigma} w_t^{-1/\sigma} \alpha_{M1}^{\beta_1/\sigma} n_{M1}^{\beta_1(1-\alpha_{M1})/\sigma\alpha_{M1}} \alpha_2^{\beta_2/\sigma} n_2^{\beta_2(1-\alpha_2)/\sigma\alpha_2}$

(53) $\quad p_{S1t} \quad = \quad w_t \alpha_{S1}^{-1} n_{S1}^{-(1-\alpha_{S1})/\alpha_{S1}}$

(54) $\quad p_{S2t} \quad = \quad w_t \alpha_2^{-\omega} n_2^{-\omega(1-\alpha_2)/\alpha_2}$

(55) $\quad p_{St} \quad = \quad w_t \alpha_{S1}^{-\theta} n_{S1}^{-\theta(1-\alpha_{S1})/\alpha_{S1}} \alpha_2^{-\omega(1-\theta)} n_2^{-\omega(1-\theta)(1-\alpha_2)/\alpha_2}$

(56) $S_{1t} \;=\; \theta w_t^{-1}\alpha_{S1} n_{S1}^{(1-\alpha_{S1})/\alpha_{S1}}$

(57) $S_{2t} \;=\; (1-\theta)w_t^{-1}\alpha_2^{\omega} n_2^{\omega(1-\alpha_2)/\alpha_2}$

(58) $Y_{St} \;=\; w_t^{-1}\alpha_{S1}^{\theta} n_{S1}^{\theta(1-\alpha_{S1})/\alpha_{S1}} \alpha_2^{\omega(1-\theta)} n_2^{\omega(1-\theta)(1-\alpha_2)/\alpha_2}$

(59) $L_{Mt} \;=\; (1-\beta_1-\beta_2)\; \tau^{-1/\sigma} w_t^{-1/\sigma} \alpha_{M1}^{-\beta_1(1-1/\sigma)} n_{M1}^{-\beta_1(1-1/\sigma)(1-\alpha_{M1})/\alpha_{M1}} \alpha_2^{-\beta_2(1-1/\sigma)}\,.$

$$n_2^{-\beta_2(1-1/\sigma)(1-\alpha_2)/\alpha_2}$$

(60) $L_{St} \;=\; (1-\omega)(1-\theta)w_t^{-1}.$

3.2.2 (a) Dynamic analysis (a) – the asymptotic steady state. Dropping the time subscript for notational ease, we denote $p_M Y_M / w$ by *Z*, which from (51)-(52) is:

(61) $Z \;=\; \tau^{-1/\sigma} w^{-1/\sigma} \alpha_{M1}^{-\beta_1(1-1/\sigma)} n_{M1}^{-\beta_1(1-1/\sigma)(1-\alpha_{M1})/\alpha_{M1}} \alpha_2^{-\beta_2(1-1/\sigma)} n_2^{-\beta_2(1-1/\sigma)(1-\alpha_2)/\alpha_2}\,.$

It is also convenient to denote $p_S Y_S / w$ (= 1/w) by *G*, and $\omega(1-\theta)$ by θ_2. With symmetry of firms in each group, it is a straightforward matter to derive the following dynamic asset pricing equations, as well as the equation for L_r:

(62) $\dot{v}_{S1} \;=\; \rho v_{S1} - \theta(1-\alpha_{S1})Gw/n_{S1}$

(63) $\dot{v}_{M1} \;=\; \rho v_{M1} - \beta_1(1-\alpha_{M1})Zw/n_{M1}$

(64) $\dot{v}_2 \;=\; \rho v_2 - (1-\alpha_2)(\theta_2 G + \beta_2 Z)w/n_2$

(65) $L_r \;=\; 1 - [(1-\theta-\theta_2) + \theta\alpha_{S1} + \theta_2\alpha_2]G - [(1-\beta_1-\beta_2) + \beta_1\alpha_{M1} + \beta_2\alpha_2]Z$

Consider first a regime in which all $\dot{n}_k$'s (*k* = *S*1, *M*1, 2) are strictly positive: from (28) the $v_k n_k$'s will then be equal to each other (= *wa*), and hence so also are their rates of change $\dot{v}_k / v_k + \dot{n}_k / n_k$. From (23)-(24) the $\dot{n}_k / n_k$'s will sum to L_r / a. Using (62)-(65) and earlier equations, it is then a straightforward matter to solve for the individual $\dot{n}_k / n_k$'s as linear functions of the 'state' variables *G* and *Z*. The same solution procedure applies for other regimes: if any one $\dot{n}_k$ is 0, only the other two $v_k n_k$'s, and their rates of change, need be equal to each other, and only these two $\dot{n}_k / n_k$'s will sum to L_r / a, while if only one $\dot{n}_k / n_k > 0$ then it simply equals L_r / a. (It should be noted, too, that the asset pricing equations (62)-(64) will hold irrespective of whether each associated $\dot{n}_k$ is positive or zero: however, as an important technical point, upon dividing any $\dot{v}_k$ by v_k, we should only set $v_k n_k$ = *wa* on the right-hand side if the associated $\dot{n}_k > 0$.)

There is thus a total of 7 possible regimes. To avoid an unduly taxonomic discussion, which does not yield significant additional economic insight, we confine ourselves to a set of parameter values, specified below, which generate a *Continuing Absolute Growth* (*CAG*) path: a path along which all three (sub-)sectors – Y_M, S_1, and S_2 – exhibit continuing growth, as a long-run outcome, *in absolute terms*. This is clearly a minimal, economically plausible restriction, and at the same time does not preclude the possibility of the value of a particular (sub-)sector's output, relative to that of another (sub-)sector, declining to 0 in the limit. We first study the regime in which only $\dot{n}_2/n_2$ and $\dot{n}_{S1}/n_{S1}$ are positive, and will show that this is the only possible limit regime exhibiting *CAG*. Subsequently we work backwards to discuss the possible approaches to this limit path.

Applying the solution procedure outlined earlier, results for key variables in this regime, termed *Regime 1* ($\dot{n}_2/n_2,\ \dot{n}_{S1}/n_{S1} \geq 0;\ \dot{n}_{M1}/n_{M1} = 0$), are:[18]

(66) $\dot{n}_{S1}/n_{S1} = (1/2a)\{1-[1-2\theta(1-\alpha_{S1})]G-[1-\beta_1(1-\alpha_{M1})]Z\}$

(67) $\dot{n}_2/n_2 = (1/2a)\{1-[1-2\theta_2(1-\alpha_2)]G-[1-\beta_1(1-\alpha_{M1})-2\beta_2(1-\alpha_2)]Z\}$

(68) $\dot{G}/G = (1/2a)\{G+[1-\beta_1(1-\alpha_{M1})]Z-1\}-\rho$

(69) $\dot{Z}/Z = -\{(1/\sigma)(\rho+1/2a)+(1/2a)(1-1/\sigma)\beta_2(1-\alpha_2)/\alpha_2\} +$
$\{(1/2\sigma)-(1-1/\sigma)[\beta_2(1-\alpha_2)/\alpha_2][\theta_2(1-\alpha_2)-1/2]\}G/a + \{(1/2\sigma)[1-\beta_1(1-$
$\alpha_{M1})] - (1-1/\sigma)[\beta_2(1-\alpha_2)/\alpha_2][(\beta_2(1-\alpha_2))-0.5(1-\beta_1(1-\alpha_{M1}))]\}Z/a$

Equations (66) and (67) are not constrained in the above, but this regime only applies in that region in G – Z space in which $\dot{n}_{S1}/n_{S1}$ and $\dot{n}_2/n_2$ are non-negative, and due cognizance of this is taken below. The corresponding equations for the other six regimes are provided in Appendix A.1 noting that cognizance will in each case have to be taken of its respective region of applicability.

Equations (68)–(69) form a self-contained dynamic system in G and Z, which are best analyzed after multiplying (68) by G and (69) by Z. Using the facts that $G = 1/w$ and that in this regime $w = v_{S1}n_{S1}/a = v_2 n_2/a$,[19] as well as the definition of V (equation (29)), it follows that if Regime 1 is to prevail finally, with strictly positive

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[18] $\dot{G}/G$ simply equals $-(\dot{v}_k/v_k+\dot{n}_k/n_k)$ for any k for which $\dot{n}_k/n_k > 0$, and $\dot{Z}/Z$ is obtained by using (61).

[19] Note that these imply that, since the n_i's and after $t = 0$ the v_i's are continuous functions of t, G (and, from (61), Z) have also to be continuous in t after $t = 0$.

growth, G has to converge to a strictly positive and finite value: a zero limiting value of G implies that V rises without bound, which can be shown to violate the transversality condition, and an unbounded limiting value of G – entailing a zero limiting value of w – implies from (65) that L_r will at some point fall to 0, so that innovation and growth will cease. With G converging to a strictly positive finite limit, and assuming the same of $\dot{n}_2 / n_2$ (as is verified immediately below), (61) shows that Z will converge to 0 in the limit. Substituting this and $\dot{G}/G$ = 0 in (68), the solution for the limiting value of G, G^*, is:

(70) $G^* \;=\; 2a\rho + 1,$

and the condition for the saddle-path stability of the limiting steady state is then easily calculated to be

(71) $\theta_2(1-\alpha_2) \;>\; \rho[2\rho + (1/a)]^{-1}.$

From (67) and (70), this is identical to the condition that $\dot{n}_2 / n_2$ converges to a strictly positive finite limit. Recalling that $\theta_2 = \omega(1-\theta)$, and using (58) and earlier equations, we see that (71) is more likely to be satisfied if the weight of D_{S2} in S_2, and of S_2 in Y_S, are high, and α_2 is low, all of which are intuitively reasonable.

The phase diagram in G – Z space may now be set up. From (68)-(69) the $\dot{G}/G = 0$ locus is negatively-sloped, while the $\dot{Z}/Z = 0$ locus (a) could have any non-zero slope and either a strictly positive or strictly negative horizontal intercept, or (b) could be horizontal with a strictly positive or negative vertical intercept, or (c) could fail to exist, in which case $\dot{Z}/Z$ is always negative. In all cases, the saddle-path will be negatively-sloped, and we depict one illustrative case in Fig 1 below.

Condition (71) implies that if the $\dot{Z}/Z = 0$ locus has a positive horizontal intercept, it must lie to the right of the horizontal intercept of the $\dot{G}/G = 0$ locus, as drawn: moreover, the $\dot{Z} = 0$ locus is the $\dot{Z}/Z = 0$ locus *as well as* the horizontal axis (Z = 0). Thus, as drawn the saddle-path will indeed converge to point A. Moreover, all along the saddle-path *in Regime 1* $\dot{n}_2 / n_2$ will be strictly positive: condition (71) ensures that $\dot{n}_2 / n_2 > 0$ in the limit, and should the saddle-path cut the $\dot{n}_2 / n_2 = 0$ locus earlier, $\dot{G}/G$ and $\dot{Z}/Z$ would from (61) have to have the same sign at that point, contrary to the (negative) slope of the saddle-path.

Results are different regarding $\dot{n}_{S1}/n_{S1}$. For a *CAG* path to prevail finally, we also require that $\dot{n}_{S1}/n_{S1}$, given by (66) above, be strictly positive in the limit, and it is easily shown that this will occur if $\theta(1-\alpha_{S1})$ obeys the same lower bound as $\theta_2(1-\alpha_2)$ ((71) above). If the $\dot{n}_{S1}/n_{S1}$ = 0 locus is negatively sloped, its horizontal intercept will lie to the right of that of the $\dot{G}/G=0$ locus, as drawn in Fig 1 (and $\dot{n}_{S1}/n_{S1}$ > 0 below it): if it is positively sloped, its horizontal intercept will be negative, and the results are qualitatively identical. In both cases, as well as when the $\dot{n}_{S1}/n_{S1}$ = 0 locus is horizontal, its vertical intercept is *below* that of the $\dot{G}/G=0$ locus, so as we move back and up the saddle-path, the condition $\dot{n}_{S1}/n_{S1} \geq 0$ will be violated at some point. The implications of this are examined shortly.

From (66)-(67), (70), and the fact that $Z \to 0$, $\dot{n}_2/n_2$ will asymptotically converge to $\theta_2(1-\alpha_2)[2\rho+(1/a)]-\rho$, positive from (71), while $\dot{n}_{S1}/n_{S1}$ converges to this same expression with $\theta_2(1-\alpha_2)$ replaced by $\theta(1-\alpha_{S1})$. Wolff's finding (*fn.* 5 above) that 'there are extreme differences in amenability to productivity growth' across service industries clearly suggests $\alpha_{S1}<\alpha_2$, while Table 3.2 of *AIM* (after removing the US government's share of GDP, taken from Congressional Budget Office (2002), from *AS*) shows that the shares of *PS* and *AS* are very close, implying $\theta \cong 0.5$. *AS* are of course significantly more labour-intensive[20] ($\omega<1$), and all these taken together imply $\theta(1-\alpha_{S1})>\theta_2(1-\alpha_2)$, so that asymptotically $\dot{n}_{S1}/n_{S1}>\dot{n}_2/n_2$. From equations (52) and (56)-(57), strict positiveness of $\dot{n}_2/n_2$ and $\dot{n}_{S1}/n_{S1}$ in the limit generates a *CAG* path, and the proof that this is the only possible *CAG* path is relegated to a footnote.[21]

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[20] Unfortunately, data on these sub-sectors' labour shares in the value of total output does not appear to be available, but certainly the data on their shares in value-added (calculated for industries for which the required data for 2000 is available in the US *National Economic Accounts*) indicates this. One might expect that, as service industries, value-added would tend to be a large component of their total output.

[21] The only other possibility for existence of such a path is if $\dot{n}_{M1}/n_{M1}>0$ in the limit as well, in which case we are in Regime 2 in Appendix A.1. However, from (61) we must have $Z \to 0$ along a *CAG* path, and equation (A4) in the Appendix then shows that $G \to 3a\rho+1$ in Regime 2: these imply from (A2) that $\dot{n}_{M1}/n_{M1}$ converges to a *negative* value, so Regime 2 clearly cannot prevail finally. Intuitively, *G* converges to (differing) fixed values in Regimes 1 and 2, and n_2 rises continually along a *CAG* path: these imply from (51)-(52) that $p_{Mt}Y_{Mt}/p_{St}Y_{St}$ will continually fall along a *CAG*

We summarize the foregoing results in

Proposition 1 *A CAG path will prevail finally if* $\theta_2(1-\alpha_2)$ *and* $\theta(1-\alpha_{S1})$ *each exceeds* $\rho[2\rho+(1/a)]^{-1}$. *In the limit* $G \to [2a\rho+1]$ *and* $Z \to 0$. *In Regime 1, the unique convergent saddle-path is negatively-sloped in G – Z space, and asymptotically* $\dot{n}_{S1}/n_{S1} > \dot{n}_2/n_2$ *if* $\theta(1-\alpha_{S1}) > \theta_2(1-\alpha_2)$.[22]

Henceforth, we assume that the conditions for existence of a *CAG* path are satisfied.[23] Next, *LPG* and *MFPG* are evidently equal in our model, and given in the limit (when input prices are constant) by the negative of the rates of change of p_M, p_{S1}, and p_{S2} (equations (51), (53)-(54), these output prices being equal to their respective unit production costs). Then, with $\omega < \beta_2$ (*fn.* 24) our result that $\dot{n}_{S1}/n_{S1} > \dot{n}_2/n_2$ implies that asymptotically *MFPG* will be highest in S_1, then *M*, then S_2 – which clearly has a bearing on the discussion in Sections 1 and 2.[24] However, we postpone a

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path, owing to non-homotheticity. As such, beyond some point, even after allowing for the spillover benefit from current n_{M1} to further innovation in *M*1, the gain from further innovation in *M*1 will not cover the research (labour) cost entailed, which rises in the limit as fast as $p_{St}Y_{St}$ (remembering that w_t (= $1/G_t$) is measured, and stabilizes, in terms of the *numeraire* $p_{St}Y_{St}$).

[22] The $1/a$ term in the asymptotic values of $\dot{n}_2/n_2$ and $\dot{n}_{S1}/n_{S1}$ given earlier is actually $\bar{L}/a$, where $\bar{L}$ is the economy's total labour force, so our model is characterized by scale effects. Since we do not vary $\bar{L}$ in our analysis, this is not an essential issue: it may also be noted that Dinopoulos and Thompson (2000) and others following them have suggested that scale effects can if necessary be conveniently eliminated by (in our notation) multiplying the parameter *a* by $\bar{L}$.

[23] Weighting the sub-sectoral output growth rates by their shares in total output, it follows that as the economy moves closer to the final position *A* in Fig 1 the growth rate of aggregate real output will be approximately constant, notwithstanding that the (constant) asymptotic sub-sectoral output growth rates can well be different (an analogous argument is made by Ngai-Pissarides). For a more detailed discussion of reconciliation between the 'Kaldor facts' and the 'Kuznets facts', see the papers by Ngai-Pissarides and Acemoglu-Guerrieri: the former, as well as Kongsamut *et. al.*, have at the same time cited various doubts regarding the empirical robustness or accuracy of all the Kaldor facts. We may additionally note from equations (52) and (58) that it is entirely possible for the asymptotic growth rates of real *M* and *S* to be closer to each other than their growth rates in nominal terms.

[24] In fact, even if $\alpha_{S1} = \alpha_2$ and $\theta = \theta_2$, so that $\dot{n}_{S1}/n_{S1} = \dot{n}_2/n_2$ in the limit, *MFPG* will still be highest in S_1, then *M*, then S_2, provided $\omega < \beta_2$ (< 1). ω is lower the higher is labour's share in S_2 – the essence of Baumol's asymptotic stagnation insight – while as indicated earlier S_1 depends heavily on *IT* inputs; empirical findings (Section 3.2.2.(c) below, including *fn.* 27) are also consistent with these inequalities. Generally, our model reveals that measured sub-sectoral *MFPG* depends in Regime 1

fuller discussion of economic implications till after our study, next, of the dynamics of the transition to this final path.

3.2.2 (b) Dynamic analysis (b) – transition to Regime 1

The initial condition of the model needs to be specified with some care: from (61), since G (= 1/w) can jump at time 0, so can Z. However, the jumps in G and Z are not independent. At time 0, (61) can be expressed as

$$(61') \quad Z_0 \;=\; \tau^{-1/\sigma} G_0^{1/\sigma} \alpha_{M1}^{-\beta_1(1-1/\sigma)} n_{M10}^{-\beta_1(1-1/\sigma)(1-\alpha_{M1})/\alpha_{M1}} \alpha_2^{-\beta_2(1-1/\sigma)} n_{20}^{-\beta_2(1-1/\sigma)(1-\alpha_2)/\alpha_2} .$$

Thus the initial condition is representable as an increasing concave function (not drawn) in Fig 1, commencing from the origin: moreover, the higher the initial value $n_{M10}^{-\beta_1(1-1/\sigma)(1-\alpha_{M1})/\alpha_{M1}} n_{20}^{-\beta_2(1-1/\sigma)(1-\alpha_2)/\alpha_2}$ (= N_0 say), the higher is Z_0 for each possible G_0. If at time 0 the economy had already attained a high productive level in M, implying a low value of N_0, the initial condition locus would be fairly flat, and the saddle-path commencing from the appropriate point on it would belong entirely to Regime 1, in which (with tastes being non-homothetic) innovation focuses more on S. Failing this, transitions from other regimes to Regime 1 are possible, and we explore this next.

Detailed study of the resulting dynamics is an algebraically fairly intricate backward induction exercise, and we summarize the results in the following Proposition, the proof of which is in Appendix A.2. For ease of reference, the bracketed terms after each regime indicate the input sets experiencing positive *R&D* in that regime: we also note that how 'far back' the system will go along any particular path in the Proposition will depend on the position of the initial condition locus.

Proposition 2 *Proceeding backwards from the final Regime 1 (S1,2) characterized in Proposition 1, the saddle-path trajectory will either*

(a) transition to Regime 2 (M1,S1,2), followed by either Regime 3 (M1,2) or 4 (M1,S1): in the latter case, it will then transition to Regime 6 (M1), and in the former case it will either remain in Regime 3 or transition to Regime 6; or

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on (a) the 'intrinsic' productivity parameters α_{S1} and α_2, (b) β_2 and ω – the importance of input set 2 in the production of M and S_2, and (c) θ – the share of S_1, *vis-à-vis* that of S_2, in service sector output, which moreover under non-homotheticity increases its share in total consumption over time: collectively, these affect the returns to *R&D* in the two input sets and the impact of *R&D* on unit costs.

(b) transition to Regime 5 (2), and either remain there or transition to Regime 3 (M1,2) followed possibly by a transition to Regime 6 (M1).

While Proposition 2 appears taxonomic, it has a clear economic logic. From the second right-hand terms of equations (62)-(64), the profits yielded at any t by an input in set k (k = M1, 2, S1) depend *inter alia* on the production parameters β_i, ω, α_k, on the taste parameter θ, and on the evolution of consumer spending on Y_S *vis-à-vis* Y_M under non-homotheticity. If the economy is initially poor, consumer spending will be concentrated on Y_M, and innovation in set M1 or 2 or both, depending on the production and taste parameters, will be profitable. As the economy develops, consumer spending progressively shifts, in relative terms, to Y_S, and at some point innovation in set S1 becomes profitable. At low expenditure levels, there will thus be no innovation in S1, and at high expenditure levels there will be no innovation in M1: given our parameter restrictions, at high expenditure levels there will definitely be continuing innovation in S1 and 2, with innovation in 2 occurring either at low expenditure levels or commencing somewhere along the way.

3.2.2 (c) Implications of the dynamic analysis

Propositions 1 and 2, in conjunction with equations (51), (53) and (54), give rise, as mentioned in the Introduction, to an interesting 'stages of growth' sequence, the stages being the *Industrial Stage*, the *Transition to the Post-Industrial Stage*, and the *Post-Industrial Stage*. At the early, Industrial Stage, p_M declines fastest, and p_{S2}/p_{S1} will also decline (when $\dot{n}_2 / n_2$ becomes positive).[25] Subsequently, p_{S1} will decline faster than p_{S2} but slower than p_M, this being the Transition stage. Finally, p_{S1} will decline even faster than p_M (which always declines faster than p_{S2}, even when $\dot{n}_{M1} / n_{M1} = 0$, since $\beta_2 > \omega$: note the crucial role played by partial overlap of variety sets, since otherwise innovation benefiting M will completely cease beyond some point), and the economy is then in the Post-Industrial Stage (which corresponds to our asymptotic steady state). This is precisely the empirical pattern we have identified in Section 2, with Wholesale and Retail Trade and FIRE first overtaking other service industries in productivity growth, and, in recent years, overtaking M as well.

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[25] The only exception to this is when there occurs a transition to Regime 4 in Proposition 2(a), which we have described below (Appendix A.2) as implausible.

An intriguing aspect of the analysis is that, owing to the induced innovation, as the relative demand for S_1 rises over the long run its relative price *decreases* – analogous to Acemoglu's (2002) result that with skill-biased technical change the relative wage of skilled labour can increase when it becomes relatively more abundant. Generally, our model's ability to generate the major, empirically relevant structural changes described above, as well as some described below, and relate them to fundamental causative factors – namely the interaction of the productivity, θ and non-homotheticity parameters – are, we believe, major advantages of the analysis.

Some further observations on the empirical applicability of the analysis are apposite. First, our result that the share of *M* in total consumer spending declines asymptotically to 0 may appear somewhat extreme. To the extent that some manufactured goods have the same expenditure-elasticity of demand as *S*, one might assume that they can be consistently aggregated with the latter in a Cobb-Douglas sub-utility function, in which case their share of total spending would remain positive, as possibly (depending on productivity parameters) would the innovation rate for inputs specific to these goods. Empirical research would serve to identify such goods, based on estimated elasticity and productivity parameters.

Second, to what extent is our asymptotic steady state, with innovation concentrated on inputs largely but not exclusively used by *S*, a reasonable approximation to reality? In *The 2005 R&D Scoreboard*, the UK's Department of Trade and Industry pointed out (*p.* 10) that the USA 'has a major presence in pharmaceuticals, *IT* hardware and software (where it has 85% of software sector *R&D*)'. *IT* hardware *R&D* would fall largely under *ICTPM* (Section 2 above), which is not part of our *M* sector, while software *R&D* occurs in both *ICTPS* and *ICTUS* (Gallaher *et. al.* (2005), *p.* 3-6). Pharmaceutical *R&D* is of course integrally associated with the income-elastic demand for health services (just as *ICTPM* and *ICTPS* provide inputs to other sectors, particularly S_1). Even on a global basis, the *Scoreboard* points out (*p*. 88) that the highest '*R&D*-intensity' groups (measured by the ratio of *R&D* expenditures to sales) consist of the pharmaceuticals and biotechnology, health, *IT* hardware, software and computer services, and electronics and electrical sectors.[26] Gallaher *et. al.* make some

[26] One puzzle is the low rates of LPG in health – amounting to -2.2% in 1990-95 and -0.3% in 1995-2000 in the US according to Tables 3A.1-2 of *AIM*, despite its high *R&D* intensity. This could reflect the different, more difficult nature of *R&D* in

interesting observations (*p*. 2-3): 'It is widely accepted that a significant share of *R&D* conducted in the manufacturing sectors supports the provision of products and services provided by the nonmanufacturing sectors. Service-sector industries' increased reliance on information technology is a prime example…In a study by Amable and Palombarini (1998), when assessing both direct and indirect *R&D*, some service sectors incorporate as much or more *R&D* into their products and services as compared with manufacturing industries.' Thus, it appears that our limiting steady state is a reasonable approximation to reality (reference may also be made to the low productivity growth rates in *M* after 1995 recorded in Table 1 earlier[27]), although it should be recognized that empirically *R&D* in M1 will continue in some industries, having expenditure-elasticities close to that of *S*.

The model also sheds light on other empirical regularities. From equations (59)-(60) and Proposition 2 it is easily seen that taste non-homotheticity ($\sigma > 1$) is necessary and sufficient for L_{St}/L_{Mt} to rise throughout the growth process. This result may be understood in the light of the counterfactual, namely $\sigma = 1$: in that case, from (52) and (57) Y_M will grow faster than S_2 simply on account of the productivity growth differential (with accompanying relative price adjustments), but the effect of this on the derived demand for L_M will be exactly offset by the same differential. If $\sigma > 1$, however, Y_M, and hence L_M, will grow slower. Non-homotheticity is also necessary and sufficient to account for the behaviour of nominal output shares, with $p_{St}Y_{St}/p_{Mt}Y_{Mt}$, $p_{S1t}S_{1t}/p_{Mt}Y_{Mt}$, and $p_{S2t}S_{2t}/p_{Mt}Y_{Mt}$ all rising continually: this follows from the fact that σ is also the inverse of the Frisch (λ_t-constant) price-elasticity of demand for C_{Mt}, and the price of *M* in terms of our *numeraire* falls continually (the corresponding elasticity for C_{St} and its constituents S_{1t} and S_{2t} is of course unity). The behaviour of the real output shares of *M*, S_1 and S_2 is less clear-cut, and depends on

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health, as well as increasing, insurance-related, administrative overheads (Krugman, 2007, basing in part on a McKinsey report), both being compensated for by the rising price of medical services. See also *fn*. 30 below. Education – like health, a labour-intensive activity that forms part of *AS* – also recorded (modestly) negative LPG during 1990-2000, but both education's and health's calculated LPG rates may also have been affected by measurement difficulties, associated with Varian's observation (*fn*. 4 above) that these are 'examples where customers tend to perceive that more labour is associated with higher quality'. In any event, this discussion does not affect the point being made above regarding the declining role of *R&D* specific to *M*.

[27] While these productivity growth rates are low, they are still above that of *NICTS* (our *AS*), consistent indeed with our specification that $\beta_2 > \omega$.

parameter values. In the asymptotic steady state, S_2 will grow faster than M if $\sigma > \beta_2 / \omega$, and *vice versa*. In other words, S_2 will grow faster if the taste bias in its favour outweighs the productivity bias in favour of M (β_2 and ω influence the growth of p_M and p_{S2} in the long run, and $1/\sigma$ and unity are the respective elasticities of response). S_1 will however grow faster than both in the long run. The fact that S as a whole can grow faster than M is consistent with the post-1960 evidence provided by Buera and Kaboski (2009), which, they point out, a model with Stone-Geary preferences and exogenous MFPG cannot adequately account for.[28] Taken as a whole, our results shed light on notable empirical regularities, including significant divergences of the price and output trends of *PS* from those of *AS*, and of each from *M*, thus underscoring the importance of analytically distinguishing between these major (sub-)sectors.

4 THE SOCIAL PLANNER'S PROBLEM

4.1 *Model specification*

Barro and Sala-i-Martin (*pp.* 227-228) and others have identified two distortions in the decentralized economy (*DE*), namely, the monopoly pricing of intermediate goods, and the fact that researchers do not internalize the intertemporal spillover effect – the fact that inventions today make subsequent inventions less costly. With transitional dynamics absent from the one-final-good models of *RGH*, the major difference between the private and socially optimal growth paths is a quantitative one,

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[28] Buera and Kaboski point out (*p.* 474) that under Stone-Geary preferences (which Kongsamut *et. al.* (2001) also employ) 'the endowments and subsistence requirements are most important at low levels of income…(there is) little income effect late in the sample'. This limitation does not apply to our utility function specification, which we view as a major advantage. Ngai and Pissarides (2007) assume homothetic tastes and (usually) price-inelastic service demand, as well as slower MFPG in *S* than *M*. A difficulty is that while this set of assumptions implies, quite realistically, that 'real consumption shares vary a lot less over time than nominal consumption shares' (*ibid.*, p. 430), it also implies (their equation (20)) that the real consumption share of services will gradually decline over time, contrary to the time-series evidence (see, for example, Kongsamut *et. al.* (2001). If instead unitary price-elasticity of service demand (and a constant investment-income ratio over time) are assumed, their model generates a constant employment share in *S*, which is also counterfactual. Lastly, if Ngai-Pissarides were to assume price-elastic service demand, they would be able to generate a rising share of *S* in nominal consumption only if MFPG in *S* exceeds that in *M*, but then the real share of *S* would rise faster than the nominal share.

the former having a lower growth rate. Our model's basic features of non-homotheticity and partially overlapping input sets result, however, in even the phases through which the economy optimally evolves being *qualitatively* different from those in *DE*. In addition, we encounter a new form of path dependence that may be termed 'Initial Condition Dependence': the steady state of the social planner (*SP*) economy is *not* independent of one of the initial conditions of the model, *even though* the latter do not affect the qualitative features of the economy's transition path. A study of the *SP* problem is thus of considerable additional interest.

Given symmetry of the intermediate inputs in each (sub-)sector and input set, efficiency requires that at each t $x_{M1jt} = x_{M1t}$, say, for all j (x_{M1t} to be determined), and likewise for x_{S1jt}, x_{M2jt}, and x_{S2jt}. D_{M1t} will then equal $n_{M1t}^{1/\alpha_{M1}} x_{M1t}$ and correspondingly for the other input sets, and equations (1), (4), (5), and (7) are modified accordingly. Dropping the time subscripts, the Hamiltonian H_S for the *SP* problem is then

$$(72)\quad H_S = \frac{[A_M L_M^{1-\beta_1-\beta_2} n_{M1}^{\beta_1/\alpha_{M1}} x_{M1}^{\beta_1} n_2^{\beta_2/\alpha_2} x_{M2}^{\beta_2}]^{1-\sigma} - 1}{1-\sigma} +$$

$$\tau \ln[A_S n_{S1}^{\theta/\alpha_{S1}} x_{S1}^{\theta} A_{S2}^{1-\theta} L_S^{1-\theta-\theta_2} n_2^{\theta_2/\alpha_2} x_{S2}^{\theta_2}] + q_{S1} a^{-1} n_{S1} L_{rS1} + q_{M1} a^{-1} n_{M1} L_{rM1} +$$

$$[q_2 a^{-1} n_2 + \phi]\ [1 - L_M - L_S - n_{S1}x_{S1} - n_{M1}x_{M1} - n_2(x_{M2} + x_{S2}) - L_{rS1} - L_{rM1}]$$

The first two square-bracketed expressions are obtained by replacing C_M and C_S in the consumer's instantaneous utility function by Y_M and Y_S respectively, and the q_k's ($k = S1, M1, 2$) are the co-state variables associated with the equations of motion (23): the final square-bracketed expression is obtained by substituting for L_{r2} using the labour-market-clearing condition. ϕ is the Kuhn-Tucker multiplier associated with the non-negativity constraint on L_{r2}: it turns out to be notationally convenient not to attach similar multipliers to L_{rS1} and L_{rM2}, but instead to express the corresponding first-order conditions as weak inequalities. The state variables are n_{M1}, n_{S1}, and n_2, the control variables are L_M, L_S, x_{M1}, x_{S1}, x_{M2}, x_{S2}, L_{rM1}, and L_{rS1}, and the complete set of first-order conditions, as well as some preliminary relationships, are provided in Section A of a Mathematical Appendix (*MA*) that is available upon request.

4.2 *Model solution*

Sharper insights into the nature of the optimal path may be obtained by examining whether the backward sequence of stages of development along it corresponds to that obtained in *DE* earlier. Some surprising qualitative differences emerge, and in the course of the analysis all other combinations of positive and zero $\dot{n}_k / n_k$'s will be discussed. As a preliminary we note the interesting result in *MA* that $q_{S1}n_{S1}$ has the constant solution $\tau\theta(\alpha_{S1}^{-1}-1)/\rho$ through all phases of the transition process: in a phase in which n_{S1} is rising, its shadow price q_{S1} will fall at the same rate, which one may conjecture is related to the logarithmic form of the sub-utility function for C_S.

Recalling that the final stage in *DE* was Regime 1, we examine the corresponding regime, denoted SP1, here, in which $\dot{n}_2 / n_2, \dot{n}_{S1} / n_{S1} \geq 0$ and $\dot{n}_{M1} / n_{M1}$ = 0. From the relevant first-order conditions in *MA* $\dot{n}_2 / n_2$ can only be positive when $\phi = 0$, and $\dot{n}_{S1} / n_{S1}$ can only be positive when $q_{S1}n_{S1}$ is then equal to $q_2 n_2$, while $\dot{n}_{M1} / n_{M1}$ can be 0 when $q_{M1}n_{M1} \leq q_2 n_2$. Equality of $q_{S1}n_{S1}$ and $q_2 n_2$ over a strictly positive time interval implies that their respective rates of change are equal over this interval, which from the relevant equations implies that Y_M, and hence $Y_M^{-(\sigma-1)}$ which we denote by *R*, is *fixed* in this interval, and given by

$$(73) \quad R^* \;=\; \tau[\theta(\alpha_{S1}^{-1}-1)-\theta_2(\alpha_2^{-1}-1)]/\beta_2(\alpha_2^{-1}-1).$$

For Regime SP1 to be part of the optimal solution we thus require that the square-bracketed term above be strictly positive, failing which input set 2 will dominate set *S*1 as an engine of growth. It is then shown in Section B(a) of *MA* that fixity of *R*, and of $q_2 n_2$ ($= q_{S1}n_{S1}$, fixed as just mentioned), implies that n_2 has also to be fixed: innovation in set 2 will completely cease in this Regime, even though it is permitted, and innovation will only occur in set *S*1! Moreover, if we also permit $\dot{n}_{M1} / n_{M1}$ to be nonzero, all three – $\dot{n}_{S1} / n_{S1}$, $\dot{n}_2 / n_2$, and $\dot{n}_{M1} / n_{M1}$ – can *potentially* be simultaneously nonzero only under a knife-edge restriction on parameter values, and even then *actually* $\dot{n}_2 / n_2$ and $\dot{n}_{M1} / n_{M1}$ will be zero, and only $\dot{n}_{S1} / n_{S1}$ will be nonzero.[29]

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[29] $\dot{n}_{S1} / n_{S1}$ will in this Regime, which is shown below to be the final regime in the *SP* economy, equal L_r/a, where L_r is given by the last square-bracketed expression in (72) without the last two terms in it, and after substitution we have that in this Regime,

$$(74) \quad L_r \;=\; 1-\{1+[\theta(\alpha_{S1}^{-1}-1)-\theta_2(\alpha_2^{-1}-1)]/\beta_2(\alpha_2^{-1}-1)\}\rho a/[\theta(\alpha_{S1}^{-1}-1)].$$

Analogous to Grossman-Helpman, therefore, and to (71) above, factors such as a low ρ and a low *a* are, for obvious reasons, necessary for optimal steady-state growth to be positive, and we assume that parameter values are such that $L_r > 0$ here. *Cont…*

There is thus a clear qualitative difference between the previous Regime 1 and the present SP1. As mentioned earlier, a market failure in the model is that private researchers do not internalize the fact that inventions today make subsequent inventions less costly. However, owing to non-homotheticity the *benefit* of subsequent inventions that raise productivity in *M* (jointly with *S*2) declines relative to the benefit from inventions for *S*1 as the economy grows – a consideration that the *SP* internalizes (recall also that $\theta(\alpha_{S1}^{-1}-1) > \theta_2(\alpha_2^{-1}-1))$. At low income levels, the former benefit exceeds the latter, and eventually a point is reached where the benefits at the margin are equalized – which explains *inter alia* how the fixed final value of Y_M is arrived at. Regime SP1 will turn out to be the final regime in the *SP* economy, in which only *PS* will keep on growing, but prior to this there will, as seen below, be growth in the other sub-sectors.[30] While this result may not hold absolutely (*fn.* 30), it does uncover a new insight that under non-homotheticity a *DE* might systematically undervalue innovation in some activities (S_1) *more* than in others (M, S_2), relative to the social optimum, and hence innovate in the latter more than is optimal.

The stage or stages prior to SP1 thus require that at least during part of the process Y_M be rising to its fixed final value: the latter can in turn be shown to entail a fixed value of $n_{M1}^{\beta_1(1-\alpha_{M1})/\sigma\alpha_{M1}} n_2^{\beta_2(1-\alpha_2)/\sigma\alpha_2}$, denoted $H(n_{M1}, n_2)$. By reasoning entirely analogous to that leading up to (73) we establish in *MA* (Section B(b)) the interesting

It is also of interest to compare the value of L_r in the respective final stages, namely Regime 1 of Section 3 and Regime SP1 of Section 4. The result is not self-evident, since attaining SP1 requires the elimination of two distortions, not one. It can be shown that when $\theta_2(1-\alpha_2)$ and $\theta(1-\alpha_{S1})$ are both at the lower bound given by the right side of (71), L_r in SP1 strictly exceeds L_r in Regime 1, and the difference widens as $\theta_2(1-\alpha_2)$ and $\theta(1-\alpha_{S1})$ rise separately from their lower bound, so that resources devoted to *R&D* and (the sum of) innovation rates are indeed higher in the *SP* solution.

[30] With *ICT* usage becoming increasingly pervasive over time, it is entirely possible that some 'specialized' inputs in S_1 will at some date start to become useful in *M* and S_2 as well, possibly with some adaptation. There would then be continuing productivity growth in *M* and S_2 too, although the 1995-2000 figures in Table 1 suggest that this effect would be limited. Another interesting perspective is provided by Hall and Jones (2007), who argue that as incomes rise health spending is particularly valued as it enables people 'to live longer and enjoy better lives' (*p*. 39). This might justify the invention of 'new and expensive medical technologies' (*p*. 40). In future research, one might allow for higher income-elasticity of demand for health than other services, inducing such inventions.

result that if any two of $\dot{n}_{S1}/n_{S1}$, $\dot{n}_2/n_2$, and $\dot{n}_{M1}/n_{M1}$ are strictly positive over a strictly positive time interval then *R*, and hence Y_M, cannot change during that interval. Thus, for Y_M to rise only one innovation rate can be permitted to be positive in some stage(s), and we show in *MA* (Section C) that, provided that the productivity term for set *M*1 $\beta_1(\alpha_{M1}^{-1}-1)$ exceeds both ρa and $\beta_2(\alpha_2^{-1}-1)$, proceeding backwards $\dot{n}_2/n_2$ will be positive (we denote this stage by SP5, regime numbers corresponding to those in *DE*), before which $\dot{n}_{M1}/n_{M1}$ will be positive (stage SP6). Such a sequence is indeed what one would expect under non-homothetic preferences, and at the same time the fact that innovation can only occur in one input set at a time marks a notable qualitative difference from the results in Section 4.

We have yet to ascertain the final values of n_{M1} and n_2 (entering into $H(n_{M1},n_2)$) individually, and it is here that we discover the new phenomenon of Initial Condition Dependence. We denote the given initial value of n_2 at t = 0 by n_{20}, and note that n_2 remains at this value throughout SP6. The sets of differential equations, for SP5 (in q_2n_2, n_2, and $q_{M1}n_{M1}$) and for SP6 (in $q_{M1}n_{M1}$, n_{M1}, and q_2n_2), are presented in *MA*, and reveal *inter alia* that the final value to which n_{M1} converges by the end of SP6, denoted n_{M1}^*, is a parameter in the differential equation set for SP5. In SP5 two critical conditions must be satisfied: (a) at the end of this regime q_2n_2 must exactly fall to the unchanging value of $q_{S1}n_{S1}$, so that a switch to SP1 can occur, and (b) at the beginning of this regime $q_{M1}n_{M1}$ must have fallen to equality with the value of q_2n_2 at that time (and then fall below q_2n_2), to permit the switch from SP6 to SP5 to occur, and noting also that this switch *must occur at* $n_2 = n_{20}$. Reconciling condition (b) with (a) requires that n_{M1}^* (a parameter in the differential equation set for SP5) be chosen appropriately, which, combined with the preceding point, implies that n_{M1}^* will depend on n_{20}: the steady-state values of n_{M1} and (from the fixed final value of $H(n_{M1},n_2)$) n_2 are thus not independent of this particular initial condition of the model!

In *MA* (Sections C, D) we demonstrate the foregoing argument geometrically, and we also linearize the model in the SP5 phase (characterization of the prior SP6 phase is a straightforward matter), and provide a precise algebraic solution for n_{M1}^*, clearly showing its dependence on n_{20}. The steady-state value of n_2, n_2^*, is shown there to vary positively, and in fact equi-proportionately, with n_{20}, and thus n_{M1}^* depends negatively on n_{20}. Intuitively, from equation (23) a higher n_{20} reduces the resource cost of further innovation in set 2, and it may be conjectured that a similar positive, though not necessarily equi-proportionate, relationship between n_{20} and n_2^* holds also in the nonlinear model. (23) thus provides the underlying rationale for Initial Condition Dependence in the model.[31] Finally, it is straightforwardly shown in *MA* that more than one transition between SP5 and SP6 in the nonlinear model cannot be optimal, and, summarizing, we have:

Proposition 3 *Along the optimal growth path, commencing from a low income level, R&D occurs sequentially, first in input set M*1*, followed by set S*2*, and finally set S*1*. The optimal trajectory is characterized by Initial Condition Dependence.*

In *MA* (Section E) we also study policy measures to enable *DE* to achieve the socially optimal growth path, and we briefly summarize the results here. Generalizing the results of Barro and Sala-i-Martin and others to our multisectoral setting, the two distortions identified earlier require two sets of policy measures in response – subsidies to intermediate input purchase, and to *R&D*, financed by lump-sum taxes (assumed feasible). The optimal subsidy rates to intermediate input purchase are $h_k = \alpha_k$, k = *S*1, *M*1, 2, the rates thus varying across input sets. As regards *R&D* subsidy rates, these vary across stages: proceeding backwards, in the final SP1 stage a subsidy only to *R&D* in set S1, at a constant rate, is called for, while in the SP5 and SP6 stages *R&D* subsidies to input sets 2 and M1 respectively should be granted. Moreover, consistent with the existence of transitional dynamics in SP5 and SP6, these latter subsidies will have to be time-varying, unlike the case in the expanding-

[31]Unlike other forms of path dependence (in non-chaotic systems), the initial condition here does not affect the phases the economy passes through, nor the qualitative properties of each phase: nonetheless, the final values of n_{M1} and n_2 are affected by n_{20}. The initial values of n_{M1} and n_{S1} are not similarly influential (except, as pointed out in *MA*, if the initial value of n_{M1} exceeds n_{M1}^*), for differing reasons. A higher initial value of n_{M1} would simply entail a shorter traverse in SP6 to n_{M1}^*, but not an increase in n_{M1}^*: presumably the declining output share of *M* as the economy grows renders the latter non-optimal (in contrast, set 2 is used in the production of both *M* and *S*). Next, path dependence as characterized here is not applicable to n_{S1}, since in the final SP1 phase n_{S1} rises continually.

product-variety models of Barro-Sala-i-Martin and Grossman-Helpman, with the relevant subsidy rate converging to 0 by the end of each stage. One may also expect that these time-varying subsidy rates will fall smoothly over time, although for some parameter values the possibility of non-monotonic convergence to 0 cannot be ruled out.

5 CONCLUSION

Perhaps the most significant insight yielded by our analysis is that the *IT* revolution, and earlier phases of technological change, both influence and are influenced by (sub-)sectoral evolution, and such innovation and structural change are integral features of an endogenous, dynamically evolving growth pattern. At low income levels, consumer demand tends to be focused relatively more on *M*, and innovation directed wholly or mainly towards this sub-sector is profitable. As income *endogenously rises* in response, demand shifts towards *S*, but not all service industries are equally amenable to innovation and productivity growth. Those that are – which form our *PS* sub-sector – will first overtake *AS* (whose inputs partially overlap with those in *M*) in productivity growth, and then overtake *M* as well – as has been the situation after the mid-1990's in the US.[32] Only a fairly elaborate model, which allows for the dynamic interaction of taste non-homotheticity, partial overlap of input sets, and differential sub-sectoral productivity and taste parameters, can we believe do full justice to the phases of economic evolution that advanced economies have passed through, or are likely to pass through, since the late 1940's. Our dynamic general equilibrium framework also provides assurance that this complex set of considerations can be integrated to produce a consistent and coherent pattern of growth and structural change.

We have also shown that under non-homotheticity a decentralized economy might systematically undervalue innovation in some kinds of activities (S_1) more than in other kinds (M, S_2), and innovate in the latter more than is socially optimal. There are thus significant qualitative, and not just quantitative, differences between the private and socially optimal paths, including the presence in the latter of a new, endogenous-growth-related form of path dependence which we term Initial Condition

[32] *AIM* point out that '(d)iffusion of *ICT* has taken place in Europe, but at a slower pace than in the United States, particularly during the second half of the 1990s' (*p*. 58).

Dependence. Achievement of social optimality in *DE* requires, notably, differentiated, time-varying *R&D* subsidies, to *M*1 first followed by 2, followed by a time-invariant subsidy to *S*1.

Various possibilities for future research arise. One is an extension to an open-economy context, with *PS* and *AS* corresponding to traded and nontraded services (Hsieh and Woo (2005)): alternatively, in the light of on-going developments, one could view both of these as tradable, with the North exporting *inter alia* some forms of *PS* (banking, financial services) to the South in exchange for other forms and for some forms of *AS* (services of radiologists, tutors), the latter having been rendered tradable through advances in telecommunications and the like. Other possibilities are adoption of a quality ladders framework, or of an overlapping generations framework, or the incorporation of physical and human capital (and endogenous wage differentials), all of which should yield interesting additional insights.

Appendix

A.1 Characterization of Regimes 2 – 7

Regime 2: This is given by $\dot{n}_k / n_k \geq 0$, k = *S*1, *M*1, 2, and we have:

(A1) $\dot{n}_{S1}/n_{S1} = (1/3a)\{1 - [1-3\theta(1-\alpha_{S1})]G - Z\}$

(A2) $\dot{n}_{M1}/n_{M1} = (1/3a)\{1 - G - [1-3\beta_1(1-\alpha_{M1})]Z\}$

(A3) $\dot{n}_2/n_2 = (1/3a)\{1 - [1-3\theta_2(1-\alpha_2)]G - [1-3\beta_2(1-\alpha_2)]Z\}$

(A4) $\dot{G}/G = (1/3a)\{G + Z - 1\} - \rho$

(A5) $\dot{Z}/Z = -\{(1/\sigma)[\rho + 1/3a] + (1/3a)(1-1/\sigma)[\beta_1(1-\alpha_{M1})/\alpha_{M1} + \beta_2(1-\alpha_2)/\alpha_2]\}$
$+\{(1/\sigma) - (1-1/\sigma)[\beta_2(1-\alpha_2)/\alpha_2][3\theta_2(1-\alpha_2)-1] +$
$(1-1/\sigma)\beta_1(1-\alpha_{M1})/\alpha_{M1}\}G/3a + (1/3)\{(1/\sigma) + (1-1/\sigma)[\beta_2(1-\alpha_2)/\alpha_2].$
$[1-3\beta_2(1-\alpha_2)] + (1-1/\sigma)[\beta_1(1-\alpha_{M1})/\alpha_{M1}][1-3\beta_1(1-\alpha_{M1})]\}Z/a$

Regime 3: $\dot{n}_{M1}/n_{M1},\ \dot{n}_2/n_2 \geq 0;\ \dot{n}_{S1}/n_{S1} = 0.$

(A6) $\dot{n}_{M1}/n_{M1} = (1/2a)\{1 - [1-\theta(1-\alpha_{S1})]G - [1-2\beta_1(1-\alpha_{M1})]Z\}$

(A7) $\dot{n}_2/n_2 = (1/2a)\{1 - [1-\theta(1-\alpha_{S1}) - 2\theta_2(1-\alpha_2)]G - [1-2\beta_2(1-\alpha_2)]Z\}$

(A8) $\dot{G}/G = (1/2a)\{[1-\theta(1-\alpha_{S1})]G + Z - 1\} - \rho$

(A9) $\dot{Z}/Z = -\{(1/\sigma)[\rho + 1/2a] + (1/2a)(1-1/\sigma)[(\beta_1(1-\alpha_{M1})/\alpha_{M1}) +$
$(\beta_2(1-\alpha_2)/\alpha_2)]\} + \{(1/2\sigma)[1-\theta(1-\alpha_{S1})] + (1-1/\sigma)[\beta_1(1-\alpha_{M1})/2\alpha_{M1}][1-$
$\theta(1-\alpha_{S1})] + (1-1/\sigma)[\beta_2(1-\alpha_2)/2\alpha_2][1-\theta(1-\alpha_{S1}) - 2\theta_2(1-\alpha_2)]\}G/a + \{(1/2\sigma) +$
$(1-1/\sigma)[\beta_1(1-\alpha_{M1})/2\alpha_{M1}][1-2\beta_1(1-\alpha_{M1})] +$
$(1-1/\sigma)[\beta_2(1-\alpha_2)/2\alpha_2][1-2\beta_2(1-\alpha_2)]\}Z/a$

Regime 4: $\dot{n}_{S1}/n_{S1}$, $\dot{n}_{M1}/n_{M1} \geq 0$; $\dot{n}_2/n_2 = 0$.

(A10) $\dot{n}_{S1}/n_{S1} = (1/2a)\{1-[1-2\theta(1-\alpha_{S1})-\theta_2(1-\alpha_2)]G - [1-\beta_2(1-\alpha_2)]Z\}$

(A11) $\dot{n}_{M1}/n_{M1} = (1/2a)\{1-[1-\theta_2(1-\alpha_2)]G - [1-2\beta_1(1-\alpha_{M1})-\beta_2(1-\alpha_2)]Z\}$

(A12) $\dot{G}/G = (1/2a)\{[1-\theta_2(1-\alpha_2)]G + [1-\beta_2(1-\alpha_2)]Z - 1\} - \rho$

(A13) $\dot{Z}/Z = -\{(1/\sigma)[\rho+1/2a]+(1/2a)(1-1/\sigma)\ \beta_1(1-\alpha_{M1})/\alpha_{M1}\}+$
$0.5[1-\theta_2(1-\alpha_2)][(1/\sigma)+(1-1/\sigma)\beta_1(1-\alpha_{M1})/\alpha_{M1}]G/a+\{(1/2\sigma)+(1-1/\sigma).$
$[\beta_1(1-\alpha_{M1})/2\alpha_{M1}]-(1-1/\sigma)[\beta_1(1-\alpha_{M1})]^2/\alpha_{M1}-$
$[\beta_2(1-\alpha_2)/2][(1-1/\sigma)(\beta_1(1-\alpha_{M1})/\alpha_{M1})+1/\sigma]\}Z/a$

Regime 5: $\dot{n}_2/n_2 \geq 0$; $\dot{n}_{S1}/n_{S1}$, $\dot{n}_{M1}/n_{M1} = 0$.

(A14) $\dot{n}_2/n_2 = (1/a)\{1-[1-\theta(1-\alpha_{S1})-\theta_2(1-\alpha_2)]G - [1-\beta_1(1-\alpha_{M1})-\beta_2(1-\alpha_2)]Z\}$

(A15) $\dot{G}/G = (1/a)\{[1-\theta(1-\alpha_{S1})]G + [1-\beta_1(1-\alpha_{M1})]Z - 1\} - \rho$

(A16) $\dot{Z}/Z = -\{(1/\sigma)[\rho+1/a]+(1/a)(1-1/\sigma)\beta_2(1-\alpha_2)/\alpha_2\} +$
$\{(1/\sigma)[1-\theta(1-\alpha_{S1})]+(1-1/\sigma)[\beta_2(1-\alpha_2)/\alpha_2][1-\theta(1-\alpha_{S1})-\theta_2(1-\alpha_2)]\}G/a+$
$\{(1/\sigma)[1-\beta_1(1-\alpha_{M1})]+(1-1/\sigma)[\beta_2(1-\alpha_2)/\alpha_2][1-\beta_1(1-\alpha_{M1})-\beta_2(1-\alpha_2)]\}Z/a$

Regime 6: $\dot{n}_{M1}/n_{M1} \geq 0$; $\dot{n}_{S1}/n_{S1}$, $\dot{n}_2/n_2 = 0$.

Equations (23) and (24) imply that $\dot{n}_{M1}/n_{M1}$ in this regime is given by the right-hand side of (A14), over a possibly different region in $G-Z$ space.

(A17) $\dot{G}/G = (1/a)\{[1-\theta(1-\alpha_{S1})-\theta_2(1-\alpha_2)]G + [1-\beta_2(1-\alpha_2)]Z - 1\} - \rho$

(A18) $\dot{Z}/Z = -\{(1/\sigma)[\rho+1/a]+(1/a)(1-1/\sigma)\beta_1(1-\alpha_{M1})/\alpha_{M1}\} +$
$[1-\theta(1-\alpha_{S1})-\theta_2(1-\alpha_2)][(\beta_1(1-\alpha_{M1})/\alpha_{M1})(1-1/\sigma)+1/\sigma]G/a+$
$\{(1/\sigma)[1-\beta_2(1-\alpha_2)]+(1-1/\sigma)[\beta_1(1-\alpha_{M1})/\alpha_{M1}][1-\beta_1(1-\alpha_{M1})-\beta_2(1-\alpha_2)]\}Z/a$

Regime 7: $\dot{n}_{S1}/n_{S1} \geq 0$; $\dot{n}_2/n_2$, $\dot{n}_{M1}/n_{M1} = 0$.

Equations (23) and (24) imply that $\dot{n}_{S1}/n_{S1}$ now is given by the right-hand side of (A14), over a possibly different region in $G-Z$ space.

(A19) $\dot{G}/G = (1/a)\{[1-\theta_2(1-\alpha_2)]G + [1-\beta_1(1-\alpha_{M1})-\beta_2(1-\alpha_2)]Z - 1\} - \rho$

(A20) $\dot{Z}/Z = (1/\sigma a)\{[1-\theta_2(1-\alpha_2)]G + [1-\beta_1(1-\alpha_{M1})-\beta_2(1-\alpha_2)]Z - 1\} - \rho/\sigma$

A.2 Proof of Proposition 2. Proceeding backwards and up the saddle-path in the final Regime 1 (Fig 1), either (a) $\dot{n}_{S1}/n_{S1}$ will fall to 0 while $\dot{n}_{M1}/n_{M1}$ remains at 0, or (b) $\dot{n}_{M1}/n_{M1}$ becomes positive while $\dot{n}_{S1}/n_{S1}$ is still positive. We consider first the latter possibility, which implies that Regime 1 has, proceeding backwards, given way to Regime 2 in Appendix A.1, with all $\dot{n}_k/n_k \geq 0$. We also note an important preliminary fact. At the transition point between Regimes 1 and 2, $\dot{G}/G$ (= $-(\dot{v}_{S1}/v_{S1}+\dot{n}_{S1}/n_{S1}) = -(\dot{v}_2/v_2+\dot{n}_2/n_2)$, with all $\dot{v}_k/v_k$ being continuous) will have to be continuous: should it jump discontinuously, the same will have to be true of $\dot{n}_{S1}/n_{S1}$ and $\dot{n}_2/n_2$ individually, which implies a discontinuous jump in L_{rt} (since $\dot{n}_{M1}/n_{M1}$ equals 0, and using equation (A2) can be seen to be continuous, at the transition point), violating the labour market equilibrium condition (36) (all other terms in (36) can be shown to be continuous). Thus, $\dot{n}_{S1}/n_{S1}$ and $\dot{n}_2/n_2$ will each have to be continuous as well, and so will $\dot{Z}/Z$, implying that the slope of the saddle-path at the transition point is also continuous. The same applies at any other transition point between two regimes.

Formally, then, a backward transition from Regime 1 to 2 will occur if the saddle-path cuts the $\dot{n}_{M1}/n_{M1} = 0$ locus of Regime 2 at a point at which $\dot{n}_{S1}/n_{S1}$ from either regime (which are equivalent at a point at which $\dot{n}_{M1}/n_{M1} = 0$) is positive. It is easily shown that this requires that the saddle-path cut the $\dot{n}_{M1}/n_{M1} = 0$ locus anywhere to the right of a ray from the origin given by

(A21) $Z = [\theta(1-\alpha_{S1})][\beta_1(1-\alpha_{M1})]^{-1}G$

From equations (A2) and (68) it may be seen that the horizontal intercept of the $\dot{n}_{M1}/n_{M1} = 0$ locus of Regime 2 lies strictly between the origin and the horizontal intercept of the $\dot{G}/G = 0$ locus of Regime 1 (the saddle-path in Regime 1 must lie to the right of this latter locus). Denoting the intersection point between this latter locus and the above ray by point H, it follows that the $\dot{n}_{M1}/n_{M1} = 0$ locus must pass to the right of point H, failing which the saddle-path will cut it to the *left* of the above ray. It turns out that in the negatively-sloped case, which requires $\beta_1(1-\alpha_{M1}) < 1/3$, this cannot be satisfied, while in the other cases, depending on parameter values, it can. After straightforward manipulations we obtain the following necessary lower bound on $\beta_1(1-\alpha_{M1})$ if a transition from 1 to 2 is to occur:

(A22) $\beta_1(1-\alpha_{M1}) \geq$ *max.* $[1/3,\ 1/\{3+(a\rho)^{-1}-[\theta(1-\alpha_{S1})]^{-1}\}]$

The second argument on the right exceeds 1/3 when $\theta(1-\alpha_{S1})$ is at its lower bound given by the right side of (71), falls as $\theta(1-\alpha_{S1})$ rises, and may fall below 1/3 when $\theta(1-\alpha_{S1})$ reaches a hypothetical maximum of 1. In the region where the second argument does not fall below 1/3, $\beta_1(1-\alpha_{M1})$ and $\theta(1-\alpha_{S1})$ are thus 'substitutes'. A high $\beta_1(1-\alpha_{M1})$ implies that innovation in M1 will persist longer in the growth process, and a high $\theta(1-\alpha_{S1})$ that innovation in S1 will commence earlier, and either of these will conduce to these innovation phases overlapping with each other.

We next show that, proceeding backwards into Regime 2 from the transition point, the saddle-path will remain negatively-sloped – it cannot turn rightwards, implying that, proceeding forwards now, both *G* and *Z* are falling, nor can it turn downwards, implying that *G* and *Z* are rising forwards. In the former case, a vertical line drawn at some value of *G* to the right of the hypothetical turning-point would intersect both arms of the saddle-path, once when $\dot{G}/G$ is negative, and again, at a *lower* value of *Z*, when $\dot{G}/G$ is positive: however, from (A4) $\dot{G}/G$ is an increasing function of *Z*, so such a scenario is not possible. A precisely analogous argument can be employed to exclude the latter case should the coefficient, of *G* now, in (A5) be positive or zero. If the coefficient is negative, that of *Z* can be seen to be negative also, and the $\dot{Z}/Z = 0$ locus in this case will be negatively-sloped, and will have a negative horizontal intercept. Setting up the resulting phase-diagram, it is easily shown that if the saddle-path is 'initially' negatively-sloped (as it is at the transition point from Regime 1 to 2), it will going backwards always remain so, and thus not turn down.

Proceeding backwards along the saddle-path in Regime 2, a point will be reached where $\dot{n}_{S1}/n_{S1}$ = 0, since from (A1) and (A4) the vertical intercept of the $\dot{n}_{S1}/n_{S1} = 0$

locus lies below that of the $\dot{G}/G = 0$ locus. From (A1) and (A3), if $[\theta_2(1-\alpha_2) - \theta(1-\alpha_{S1})]G + \beta_2(1-\alpha_2)Z$ is positive at that point, $\dot{n}_2/n_2$ will be positive there, and conversely. Although we have earlier argued that $\theta_2(1-\alpha_2) - \theta(1-\alpha_{S1})$ is likely to be negative, we also argued that β_2 exceeds ω and, *a fortiori*, θ_2, and moreover proceeding up the saddle-path *G* is declining and *Z* is rising. Thus, it appears plausible to suppose that $\dot{n}_2/n_2$ remains positive, and so the backwards crossover is from Regime 2 to 3: the alternative case is quite straightforward, with the backwards trajectory transiting from Regime 2 to Regime 4, whence from the resulting phase diagram it has to continue backwards in the same northwest direction, and then, upon crossing the $\dot{n}_{S1}/n_{S1}$ = 0 locus, transition to Regime 6 (analyzed below) and remaining there, all the way towards the vertical axis if necessary.

Again, it can be shown that in Regime 3 the backwards trajectory will maintain a northwest movement.[33] It will then, as it converges towards the vertical axis, either remain all the way in Regime 3, or transition to Regime 6, in which $\dot{n}_2/n_2$ is 0 and only $\dot{n}_{M1}/n_{M1}$ is positive, and remain there.[34]

Lastly, we briefly consider the case in which the first backward transition is from Regime 1 to Regime 5 – in other words, in which $\dot{n}_{S1}/n_{S1}$ falls to 0 before $\dot{n}_{M1}/n_{M1}$ becomes positive, if ever, so that only $\dot{n}_2/n_2$ is positive. It is readily shown that in Regime 5, all three loci – $\dot{G}/G = 0$, $\dot{Z}/Z = 0$, and $\dot{n}_2/n_2 = 0$ – are negatively sloped, and do not intersect in the positive quadrant, with the $\dot{n}_2/n_2 = 0$ locus being outermost, followed by the $\dot{Z}/Z = 0$ locus. Between the $\dot{G}/G = 0$ and $\dot{Z}/Z = 0$ loci, the backwards trajectory will have a northwest movement, and cannot transition

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[33] From (A9) it is not possible to have a parameter configuration such that the coefficient of *G/a* is negative and that of *Z/a* positive. (This is easily seen after dividing the entire coefficient of *G/a* by $[1-\theta(1-\alpha_{S1})]$, and remembering that $\beta_2 > \omega > \theta_2 (= \omega(1-\theta))$.) In all other cases, if the backwards crossover from Regime 2 to 3 occurs in a northwest direction, as it must if there is to be such a crossover, the resulting phase diagrams show that the backwards trajectory cannot transition to any other phase.

[34] In Regime 6, all three loci – $\dot{G}/G = 0$, $\dot{n}_{M1}/n_{M1} = 0$, and $\dot{Z}/Z = 0$ – will intersect at a common point in the positive quadrant, and will all be negatively-sloped, with the flattest (the algebraically largest) slope belonging to the $\dot{G}/G = 0$ locus, followed by the $\dot{Z}/Z = 0$ and then the $\dot{n}_{M1}/n_{M1} = 0$ loci. The backward saddle-path trajectory will thus extend all the way towards the vertical axis. It should be mentioned that there appears to exist a slight theoretical possibility that, instead of transiting backwards from Regime 3 to 6, the trajectory transits from Regime 3 to 5, in which $\dot{n}_{M1}/n_{M1} = 0$ and only $\dot{n}_2/n_2 > 0$. A sufficient condition to exclude this is $\beta_1(1-\alpha_{M1}) > 1/2$, which is quite plausible given that the lower bound in (A22) is only a necessary one. The likelihood of this slight theoretical possibility is somewhat greater if $\beta_2(1-\alpha_2)$ is close to or exceeds $\beta_1(1-\alpha_{M1})$, but this is itself highly unlikely since it would imply a rather low share of labour in Y_M. We thus ignore this possibility, also because it does not affect the Stages of Growth pattern identified in the text.

to any other region in this regime's phase diagram. Proceeding backwards, the trajectory may either remain in Regime 5 all the way towards the vertical axis, or transition to Regime 3, in which besides $\dot{n}_2 / n_2$ $\dot{n}_{M1} / n_{M1}$ also becomes positive. A necessary condition for the latter is $\beta_1(1-\alpha_{M1}) > \rho[2\rho + (1/a)]^{-1}$, which ensures that the vertical intercept of the $\dot{n}_{M1} / n_{M1} = 0$ locus of Regime 3 is above that of the $\dot{G} / G = 0$ locus of Regime 5, noting that the two regimes are equivalent at any point along the $\dot{n}_{M1} / n_{M1} = 0$ locus. (At the same time, $\beta_1(1-\alpha_{M1})$ should not be so high that the first backwards transition was from Regime 1 to Regime 2, in which case the preceding analysis applies.) Should the system cross into Regime 3, the preceding analysis then applies, from then backwards. This completes the proof of the Proposition. $\|$

References

Aarnio, O., "Can we trust the service sector to generate decent rates of growth and employment?", *Personnel Review*, 28 (1999), 382-405

Acemoglu, D., "Directed Technical Change," *Review of Economic Studies*, 69 (October 2002), 781-810

Acemoglu, D., and Guerrieri, V., "Capital Intensity and Non-Balanced Economic Growth," NBER Working Paper 12475, August 2006 (revised version published as "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy*, 116 (June 2008), 467-498)

Bagnoli, P., "Productivity Growth: Discussion and Twelve Sector Survey," *Brookings Discussion Papers in International Economics*, No. 132, April 1997

Barro, R.J., and Sala-i-Martin, X., *Economic Growth*, 2nd ed. (New York: McGraw-Hill), 2004

Basu, S., Inklaar, R., and Wang, J.C., "The Value of Risk: Measuring the Service Output of US Commercial Banks," *Economic Inquiry*, 49 (January 2011), 226-245

Baumol, W.J., "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review*, 57 (June 1967), 415-26

Baumol, W.J., Blackman, S.A.B., and Wolff, E.N., "Unbalanced Growth Revisited: Asymptotic Stagnancy and New Evidence," *American Economic Review*, 75 (September 1985), 806-817

Baumol, W.J., Blackman, S.A.B., and Wolff, E.N., *Productivity and American Leadership: The Long View* (Cambridge, MA: The MIT Press, 1989)

Baumol, W.J., "Services as Leaders and the Leader of the Services," in Gadrey and Gallouj (*eds.*), 147-63

Blow, L., "Household Expenditures Patterns in the UK," *DEMPATEM Working Paper No. 2*, Amsterdam Institute for Advanced Labour Studies, University of Amsterdam, 2004

Bosworth, B.P., and Triplett, J.E., "Services Productivity in the United States: Griliches' Services Volume Revisited," Working Paper, The Brookings Institution, 2003

Buera, F.J., and Kaboski, J.P., "The Rise of the Service Economy," Working Paper, Northwestern University, 2006

Buera, F.J., and Kaboski, J.P., "Scale and the Origins of Structural Change," Working Paper, Northwestern University, 2007

Buera, F.J., and Kaboski, J.P., "Can Traditional Theories of Structural Change Fit the Data?", *Journal of the European Economic Association*, 7 (April 2009), 469-77

Chiang, A.C., *Elements of Dynamic Optimization* (New York: McGraw-Hill, 1992)

Department of Trade and Industry (UK), *The 2005 R&D Scoreboard*, published report

Dinopoulos, E., and Thompson, P., "Endogenous Growth in a Cross Section of Countries," *Journal of International Economics* 51 (August 2000), 335–362

Echevarria, C., "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review*, 38 (May 1997), 431-52

Gadrey, J., and Gallouj, F. (*eds*.), *Productivity, Innovation and Knowledge in Services* (Cheltenham, UK: Edward Elgar, 2002)

Gadrey, J., "The Misuse of Productivity Concepts in Services: Lessons From a Comparison between France and the US," in Gadrey and Gallouj (*eds*.), 26-53

Gallaher, M., Link, A., and Petrusa, J., *Measuring Service-Sector Research and Development*, RTI International, published report, 2005

Gardes, F., and Starzec, C., "Household Demand Patterns in France 1980-1995," *DEMPATEM Working Paper No. 6*, Amsterdam Institute for Advanced Labour Studies, University of Amsterdam, 2004

Grossman, G., and Helpman, E., *Innovation and Growth in the Global Economy*, (Cambridge, MA: The MIT Press, 1991)

Guo, J., and Planting, M.A., "Using Input-Output Analysis to Measure US Economic Structural Change over a 24 Year Period," paper presented at *The 13th International Conference on Input-Output Techniques*, Macerata, Italy, Aug 21-28 2000

Hall, R.E., and Jones, C.I., "The Value of Life and the Rise in Health Spending," *Quarterly Journal of Economics*, 122 (February 2007), 39-72

Hillebrand, E.E., Lewer, J.J., and Zagardo, J.T., "Backtracking from Globalization," *Global Economy Journal*, 10(4), 2010, Article 6

Houthakker, H.S., "Additive Preferences," *Econometrica*, 28 (April 1960), 244-257

Hsieh, C.T., and Woo, K.T., "The Impact of Outsourcing to China on Hong Kong's Labor Market," *American Economic Review*, 95 (December 2005), 1673-87

Huether, D., Interview with Society for Human Resources Management, March 12 2010 (http://www.shrm.org/Research/FutureWorkplaceTrends/Pages/DavidHuether.aspx)

Jorgensen, D.W., Ho, M.S., Samuels, J.D., and Stiroh, K.J., "Industry Origins of the American Productivity Resurgence," Working Paper, June 13 2007

Klenow, P.J., "Industry Innovation: Where and Why," *Carnegie-Rochester Conference Series on Public Policy*, 44 (June 1996), 125-150

Kongsamut, P., Rebelo, S., and Xie, D., "Beyond Balanced Growth," *NBER Working Paper 6159*, September 1997

Kongsamut, P., Rebelo, S., and Xie, D., "Beyond Balanced Growth," *Review of Economic Studies*, 68 (October 2001), 869-882

Kozicki, S., "The Productivity Growth Slowdown: Diverging Trends in the Manufacturing and Service Sectors," Federal Reserve Bank of Kansas City, *Economic Review*, 1997 (QI), 31-46

Krugman, P., "The Health Care Racket," *New York Times*, February 16 2007

Lum, S.K.S., Moyer, B.C., and Yuskavage, R.E., "Improved Estimates of Gross Product by Industry for 1947-98," *Survey of Current Business*, 80 (June 2000), 24-54

Lysko, W., "Manufacturing Multifactor Productivity in Three Countries," *Monthly Labor Review*, 118 (July 1995), 39-55

McGuckin, R.H., and Stiroh, K.J., "Do Computers Make Output Harder to Measure?", *Journal of Technology Transfer*, 26 (October 2001), 295-321

Meckl, J., "Structural Change and Generalized Balanced Growth," *Zeitschrift fur Nationalokonomie*, 77 (December 2002), 241-266

Ngai, L.R., and Pissarides, C.A., "Structural Change in a Multi-Sector Model of Growth," *American Economic Review*, 97 (March 2007), 429-443

Ogaki, M., "Engel's Law and Cointegration," *Journal of Political Economy*, 100 (October 1992), 1027-46

Romer, P.M., 'Endogenous Technological Change,' *Journal of Political Economy*, 98 (October 1990), S71-S102

Schmitt, J., “Estimating household consumption expenditure in the US using the Interview and Diary portions of the 1980, 1990, and 1997 Consumer Expenditure Surveys,” *DEMPATEM Working Paper No. 1*, Amsterdam Institute for Advanced Labour Studies, University of Amsterdam, 2004

Sharpe, A., "Recent Productivity Developments in Canada and the United States: Productivity Growth Deceleration versus Acceleration," *International Productivity Monitor* 8 (Spring 2004), 16-26

Sichel, D.E., “The Productivity Slowdown: Is a Growing Unmeasurable Sector the Culprit?”, *Review of Economics and Statistics*, 79 (Aug 1997), 367-70

Stiroh, K.J., “Information Technology and the U.S. Productivity Revival: What Do the Industry Data Say?”, *American Economic Review*, 92 (December 2002), 1559-1576

Stokey, N.L., “Learning by Doing and the Introduction of New Goods,” *Journal of Political Economy*, 96 (August 1988), 701-717

US Congressional Budget Office, “A 125-Year Picture of the Federal Government’s Share of the Economy, 1950-2075” (http://www.cbo.gov/ftpdoc.cfm?index=3521&type=0&sequence=0), 2002

US Department of Commerce, Bureau of Economic Analysis, *National Economic Accounts* (http://www.bea.gov/national/nipaweb/SelectTable.asp?Selected=N#S6), 2011

US Department of Labor, Bureau of Labor Statistics, *Producer Price Indexes* (http://www.bls.gov/ppi/), 2007

Van Ark, B., Inklaar, R., and McGuckin R.H., ‘“Changing Gear”: Productivity, ICT and Service Industries in Europe and the United States’, in Christensen, J.F., and Maskell, P. (eds.), *The Industrial Dynamics of the New Digital Economy*, (Cheltenham, U.K.: Edward Elgar, 2003), 56-102

Varian, H., “Information Technology May Have Cured Low Service-Sector Productivity,” *The New York Times*, Feb 12 2004

Wildman, S.S., “Assessing Quality-Adjusted Changes in the Real Price of Basic Cable Service,” unpublished paper, Michigan State University, Sept 2003

Wolff, E.N., “How Stagnant are Services?”, in Gadrey and Gallouj (eds.), 3-25

Xu, Y., “A Model of Trade and Growth with a Non-Traded Service Sector,” *Canadian Journal of Economics*, XXVI (November 1993), 948-960

Yuskavage, R.E. (1996), “Improved estimates of gross product by industry, 1959-94,” *Survey of Current Business*, 76 (August 1996), 133-155

Yuskavage, R.E., ‘Gross Product by Industry Price Measures, 1977–96’, *Survey of Current Business*, 78 (March 1998), 17-25

Table 1 Productivity growth and GDP shares of ICT-producing, ICT-using and non-ICT industries in the US

	Productivity Growth		**GDP Share**
	1990 - 1995	**1995 - 2000**	**2000**
Total Economy	1.1	2.5	100.0
ICT-Producing Industries	8.1	10.1	7.3
ICT-Producing Manufacturing	15.1	23.7	2.6
ICT-Producing Services	3.1	1.8	4.7
ICT-Using Industries[a]	1.5	4.7	30.6
ICT-Using Manufacturing	-0.3	1.2	4.3
ICT-Using Services	1.9	5.4	26.3
Non-ICT Industries	0.2	0.5	62.1
Non-ICT Manufacturing	3.0	1.4	9.3
Non-ICT Services	-0.4	0.4	43.0
Non-ICT Other	0.7	0.6	9.8

Notes: Productivity is defined as value added per person employed.
a) excluding ICT-producing.

Source: van Ark, Inklaar, and McGuckin (2003), Table 3.1

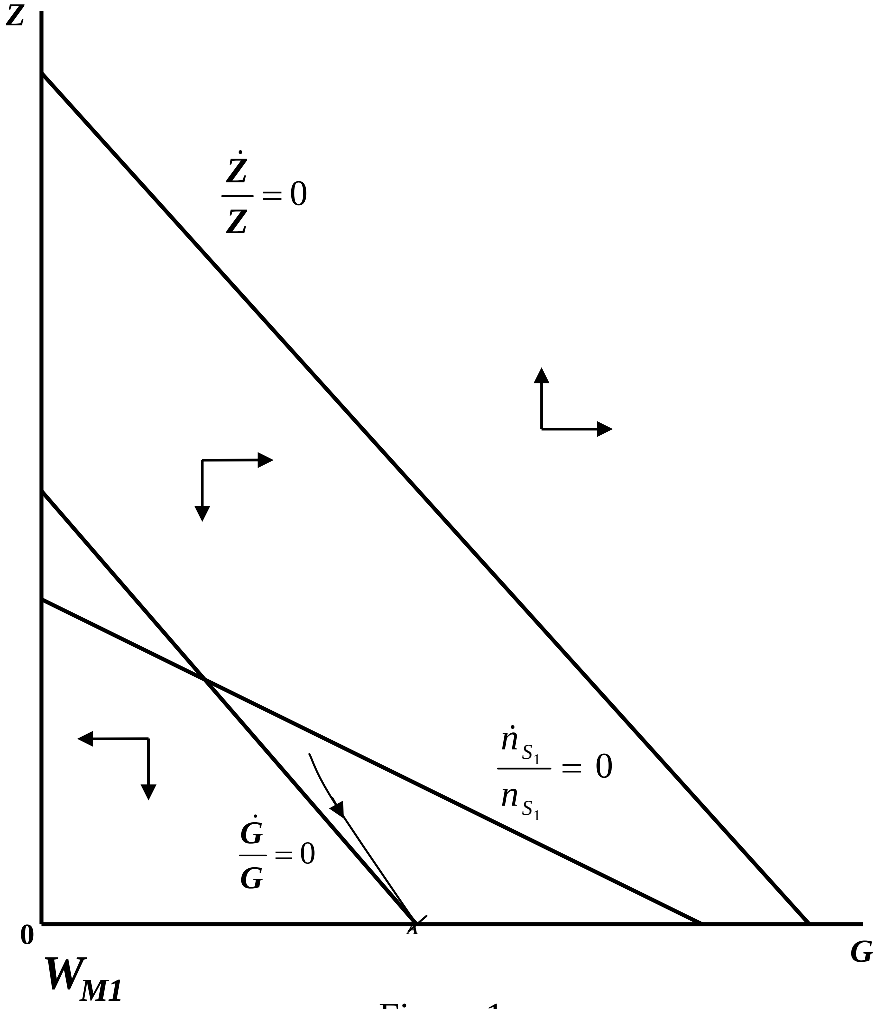

Figure 1